BS/2111

21085/NH

# Advanced Calculus-I

Sem-III (Syll-Dec-2020)

Time :- 3hrs

M.M. - 40

## SECTION-A

$2\times 6 = 12$

I (a) Prove that the function f(x, y) = $\sqrt{|xy|}$ is not differentiable at the origin. (3)

I(b) Expand by Taylor's theorem, $x^4 + x^2y^2 - y^4$ about the point (1, 1) upto the terms of the second degree. (3)

II Find all the points of maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12\cancel{x}y$$

Also discuss the saddle points (if any) of the function. (6)

III If $\alpha, \beta, \gamma$ are roots of the equation $\frac{x}{a+k} + \frac{y}{b+k} + \frac{z}{c+k} = 1$ in $k$,

prove that $\frac{\partial(x,y,z)}{\partial(\alpha,\beta,\gamma)} = \frac{-(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)}{(b-c)(c-a)(a-b)}$. (6)

IV (a) Give an example of a function of two variables in which the two repeated limits exist, but the simultaneous limit does not exist. (3)

IV (b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)^2 = \frac{9}{(x+y+z)^2}$ (3)

## SECTION-B

$2\times 6 = 12$

V (a) Evaluate $\int_0^{\pi}\int_0^{\pi} |\cos(x + y)| dxdy$. (3)

V (b) Change the order of integration in $\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} f(x, y)dxdy$ (3)

VI Prove that $\iint \sqrt{|y - x^2|}\, dx\, dy = \frac{3\pi+8}{6}$ over Area $A = [-1, 1] \times [0, 2]$. (6)

VII (a) Evaluate $\iiint \frac{dxdydz}{\sqrt{x^2+y^2+(z-2)^2}}$, over region $V = \{(x, y, z):\ x^2 + y^2 + z^2 \leq 1\}$ (3)

VII (b) Find moment of inertia of the cylinder $x^2 + y^2 \leq a^2, 0 \leq z \leq h$ with uniform mass density 1 about the z-axis. (3)

VIII (a) Find the centroid of a tetrahedron with uniform density 1 and bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$ (3)

VIII (b) Show that $\iiint \frac{dxdydz}{a^2+x^2+y^2+z^2} = \pi a(4 - \pi)$ over the region $V = \{(x, y, z): x^2 + y^2 + z^2 \leq a^2\}$ (3)

## SECTION-C

IX (a) Define saddle point.

IX (b) If z= f(x, y) is a homogeneous function of x and y of degree n, then show that

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$$

IX (c) State Young's Theorem.

IX (d) Show that the following functions are not independent of each other

$$f(x,y)=\frac{x+y}{1-xy} \quad and \quad g(x,y)=\tan^{-1}x+\tan^{-1}y\,.$$

IX (e) Evaluate $\iiint(z^5+z)\,dxdydz \quad over\ region\ V=\{(x,y,z): x^2+y^2+z^2\le 1\}$

IX (f) Evaluate $\int_0^1\int_0^1\frac{x-y}{x+y}dxdy$

IX (g) Find the area of the region bounded by the line y = x and the parabola $y^2$ = 4x.

IX (h) Define Moments of Inertia of solid of mass M continuously distributed with mass

Density $\mu(x,y,z) throughout\ a\ region\ V \subset \mathbf{R}^3$

21085/NH (2× 8 = 16)