AS|2111 (Ist Semester) 21084|NH

COORDINATE GEOMETRY

(Syll. Dec-2019)

Time: 3 Hours M. Marks: 40

PAPER III

Note :Attempt five questions in all selecting two questions each from Section A and B. Section C is compulsory.

Section A

Q.1. (a) Find the equation of the parabola with focus $(-1, 1)$ and directrix the line $x + y + 3 = 0$. Also find the latus rectum, equation of its axis and the coordinates of the vertex.

(b) Define pole and polar. Prove that the locus of the poles of tangents to the parabola $y^2 = 4ax$ w.r.t. the parabola $y^2 = 4bx$ is the parabola $y^2 = \frac{4b^2}{a}x$. **(3+3)**

Q.2 (a) Show that the locus of the middle points of chords of the parabola $y^2 = 4ax$ through the vertex is the parabola $y^2 = 2ax$.

(b) For what values of k does the equation $2x^2 + 3xy + y^2 - 3x + ky - 2 = 0$ represents a pair of straight lines? **(3+3)**

Q.3. (a) Find the equation of chord of contact of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$.

(b) Show that the straight line $7x + 6y = 13$ is a tangent to the parabola $y^2 - 7x - 8y + 14 = 0$. Find the point of contact. **(3+3)**

Q.4 (a) Prove that the tangent at the extremity of a diameter of a parabola is parallel to the chords bisected by the diameter.

(b) Show that from any point, three normals can be drawn to a parabola of which at least one must be real. **(3+3)**

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Section B

Q.5. (a) Find the equations of the tangents and normals at the points of the ellipse $x^2 + 4y^2 = 25$ whose ordinate is 2.

(b) Find the equation of the ellipse which passes through the point $(-3,1)$ and has the eccentricity $\sqrt{\frac{2}{5}}$. **(3+3)**

Q. 6. (a) If the chord of contact of tangents from (α, β) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = c^2$, show that the point (α, β) lies on the ellipse $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$.

(b) Find the point of intersection of the line $y = mx + c$ with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(3+3)**

Q.7. (a) Prove that only one of a pair of conjugate diameters of a hyperbola meets the hyperbola in real points.

(b) Chords of the hyperbola $x^2 - y^2 = a^2$ touch the parabola $y^2 = 4ax$. Prove that the locus of their middle points is the curve $y^2(x - a) = x^3$. **(3+3)**

Q.8. (a) Define a hyperbola. Find the equations of directrices, centre, foci of hyperbola $16x^2 - 9y^2 - 32x + 36y - 164 = 0$.

(b) Prove that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$. **(3+3)**

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Section C

Q.9. (a) Derive the equation of a rectangular hyperbola.

(b) State the diameter property of the parabola

(c) Find the equation of tangent and normal to the parabola $y^2 = 6x$ at the point $\left(\frac{8}{3},4\right)$.Find the equations of tangents to the ellipse $x^2 + 3y^2 = 3$ which are parallel to the line $y = 4x + 8$.

(d) Define the focus, eccentricity and directrix of an ellipse.

(e) Find the equation of the hyperbola whose asymptotes are the lines $x + 2y + 1 = 0$ and $2x + y + 3 = 0$ and which passes through the point $(1,2)$.

(f) State and prove the reciprocal property of pole and polar with respect to a parabola.

(g) Show that the tangents at the extremities of the major axis of an ellipse are parallel to the minor axis.

(h) Find the points of the parabola $y^2 = 8x$, whose ordinate is twice the abscissa.

$(8 \times 2 = 16)$