R

Roll No. ..... Total Pages : 4

13723/NH

C/2111

MATHEMATICAL METHODS–I

Option–(i)

Semester–V

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION—A

1. Find the Fourier series to represent $f(x) = x\sin x$ from $x = 0$ to $x = 2\pi$. 6

2. Find the Fourier series of the function given by :

$$f(x) = \begin{cases} x & , \text{ when } 0 < x < \pi \\ 2x - x & , \text{ when } \pi < x < 2\pi. \end{cases}$$

Deduce that : $\frac{1}{1^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{8}$. 6

3. Expand $f(x) = x - x^3$ as Fourier series in interval $-l < x < l$. 6

4. If $(x) = \begin{cases} x & , \text{ when } 0 < x < \pi/2 \\ x - x & , \text{ when } \pi/2 < x < \pi \end{cases}$, show that :

$$f(x) = \frac{4}{x}\left[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 3x}{3^2} + \ldots\right].$$ 6

SECTION—B

5. State and prove Existence theorem of Laplace transform. 6

6. (a) Find the Laplace transform of $\sin\sqrt{t}$. 6

(b) Evaluate : $\int_0^\infty t^3 t^{-1} \sin t \, dt$.

7. (a) State and prove Second Shifting theorem of inverse Laplace transform.

(b) Find the inverse Laplace transform of $\frac{1}{s^3(s+1)}$.

8. Apply Convolution theorem to find inverse Laplace transform of $\frac{1}{(s^2+a^2)^3}$. 6

SECTION—C

9. Answer the following questions briefly : 8×2=16

(i) Define complex form of Fourier series.

(ii) If m and n are the integers, then evaluate $\int_a^{a+2x} \cos nx\, dx$.

(iii) For a periodic function of period 2π, prove that $\int_{\pi}^{-\pi} f(x)dx = \int_{\pi}^{-\pi} f(c+x)dx$.

(iv) State Main theorem.

(v) Define Gamma function.

(vi) Prove that for $t \geq 0, L(1) = \frac{1}{s}$.

(vii) State cosine integral function.

(viii) Prove $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$.