

Roll No.

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SECTION—A**13722/NH****C-2111****DISCRETE MATHEMATICS-I**

Paper-II

Semester-V

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

1. (a) Find the number of integers between 1 and 10000 inclusive that are not divisible by 4, 5, 7 or 10. 3
- (b) Give a Grammar generating the language L in which every sentence is a string of equal number of a's and b's. 3
2. (a) How many different automobiles license plates made up of two letters followed by four digits are there ? 3
- (b) In how many ways can two adjacent squares be selected from an 8×8 chess board ? 3
3. Let (P, \leq) be a partially set. Suppose the length of the longest chain in P is n. Then the elements in P can be partitioned into n disjoint antichains. 6

4. (a) Let R be symmetric and transitive relation on a set A . Show that if for every a in A , there exists b in A such that (a, b) is in R , then R is an equivalence relation. 3
- (b) Construct a grammar for the language $L = \{x : x \in \{a, b\}^*, \text{ number of } a\text{'s in } x \text{ is a multiple of } 3\}$. 3

SECTION—B

5. (a) First, prove that any two multigraphs G of order 3 with degree sequence $(4, 4, 4)$ are isomorphic. Then :
- (i) Determine all the non-isomorphic induced subgraphs of G .

(ii) Determine all the non-isomorphic spanning subgraphs of G .

(iii) Determine all the non-isomorphic subgraphs of order 3 of G . 3

(b) A connected graph of order $n \geq 3$ with a bridge does not have a Hamilton cycle. 3

6. (a) A multigraph is bipartite if and only if each of its cycles has even length. 3

(b) If T is a binary tree of height h and order p , then $(h + 1) \leq p \leq 2^{(h+1)} - 1$. 3

7. Write a regular grammar for the language L over the alphabet $\{a, b\}$, where L is the set of

all strings with exactly two b's. Draw a finite-state machine to recognize this language. Have the last output be 1 if the input word is in L and 0 if it is not in L. 6

8. (a) Every circuit has an even number of edges in common with every cut-set. 3
- (b) State and prove Euler theorem. 3

SECTION—C

9. Answer the following : 8×2=16
- (i) Is the Cartesian product of two lattices always a lattice ? Prove your claim.

- (ii) How many integers between 0 and 99999 inclusive have among their digits each of 2, 5, 8 ?
- (iii) A bag contains 100 apples, 100 bananas, 100 oranges and 100 pears. If I pick one piece of fruit out of the bag every minute, how long will it be before I am assured of having picked at least a dozen pieces of fruit of the same kind ?
- (iv) A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Determine the number of vertices and edges in the tree.
- (v) Construct a grammar for the language $L = \{aaaa, aabb, bbaa, bbbb\}$.

- (vi) Show that a linear planar graph with less than 30 edges has a vertex of degree 4 or less.
- (vii) Draw a complete bipartite graph on two and four vertices.
- (viii) Show that a regular binary tree has an odd number of vertices.