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Roll No. ........................ Total Pages : 4

# 13661/NH

**B–2111**

**ANALYSIS**

Paper–I

Semester–III

Time Allowed : 3 Hours] [Maximum Marks : 40

**Note :** The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

## SECTION—A

1. Examine the convergence of the series : 3

(a) $\sum_{n=1}^{\infty} \frac{1.3.5.....(4n-5)(4n-3)x^{2n}}{2.4.6.....(4n-4)(4n-2)4n}, x > 0.$

(b) $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+....\frac{2^n-2}{2^n+1}x^{n-2}+......$ 3



2. (a) Find the interval of absolute convergence for the series $x+\frac{x^2}{2^2}+\frac{2!x^3}{3^3}+\frac{3!x^4}{4^4}+..........$ 3

(b) Show that the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ converges. 3

3. Prove that the sequence defined by $a_{n+1}=\left(\frac{1}{2}\right)\left(a_n+\frac{1}{a_n}\right), n \geq 1, a_1 = 1$ is convergent and evaluate lim $a_n$. 6

4. Evaluate the following limits : 3,3

(a) $\lim_{n\to\infty} \frac{n^n}{\{(n+1)(n+2)......(n+n)\}}.$

(b) $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n}.$

## SECTION—B

5. (a) If the function f is bounded and integrable on [a , b], then prove that $f^2$ is also integrable.

(b) Using the definition of integral as the limit of a sum, show that $\int_0^a \sin x\, dx = 1-\cos a.$ 3

6. (a) Find the upper and lower Riemann integrals for the following function defined on [0, 1].

$$f(x)=\begin{cases}(1-x^2)^2, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational.}\end{cases}$$ 3

(b) A bounded function f is integrable in [a, b], if the set of its points of discontinuity is finite. Is the converse true? Justify your answer. 3

7. Examine the convergence of Gamma function. 6

8. (a) Examine the existence of the improper integral

$\int_0^{\infty} \frac{2x^2}{x^4-1}\,dx$ and evaluate if exist. 3

(b) Show that $\int_0^{1}\left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$ is convergent.

3

9. Attempt all of the following questions : 8×2=16

(i) Let P = {0, 1, 2, 4} be a partition of the interval [0, 4]. Let f (x) = $x^2$. Find norm P, U (P, f), L (P, f).



(ii) Give an example of Riemann integrable function defined on [0, 2] and having discontinuity only at 1 and 2.

(iii) The value of $\lim_{n\to\infty}(1+1/n)^n$ lies between ............

(iv) If $\langle f_n \rangle$ is any bounded sequence, then prove that $\lim_{n\to\infty}\inf(-f_n) = -\lim_{n\to\infty}\sup(f_n)$.

(v) State Abel's and Dirichlet's tests for convergence of improper integrals.

(vi) Examine the convergence of $\int_1^{\infty}\frac{\log x}{x^2}\,dx$.

(vii) For what value of n in $\lim_{n\to\infty}\left(\frac{u_n}{u_{n+1}}-1\right)$, the series $\sum_{n=1}^{\infty} u_n$ is divergent.

(viii) Apply Cauchy's integral test to examine the convergence of $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$.