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# 13603/NH

**A–2111**

**LINEAR ALGEBRA**

Paper–III

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 40

**Note :** The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

## SECTION—A

1. By using row transformations, find the inverse of the matrix A where : 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 5 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$



2. Find the rank of the matrix : 9

$$\begin{bmatrix} 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -3 & 1 \\ 3 & -4 & -7 & -8 & -2 \end{bmatrix}.$$

3. Examine the consistency of the following equations and if consistent, determine the complete solution : 9

$x - y + 2z = 4$

$3x + y + 4z = 6$

$x + y + z = 1.$

4. Verify Cayley-Hamilton theorem for the matrix : 6

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

## SECTION—B

5. (a) Is the vector (2, –5, 3) in $V_3(R)$ a linear combination of the vectors :

(1, –3, 2), (2, –4, –1) and (1, –5, 7) ? 3

(b) The Union of two subspaces is not necessarily a subspace. Justify by examine. 3

6. Define the Finite dimensional vector space. Show that any two bases of a finitely generated vector space have the same number of elements. 6

7. Prove that the set L(V, W) of all linear transformations from V(F) to W(F) forms a vector space over the field F with respect to addition and sclar multiplication of compositions. 6

8. If the matrix of a linear operator T on $R^3$ relative to the usual basis is : 6

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}.$$

then find the matrix of T relative to the basis :

B = {(1, 2, 2), (1, 1, 2), (1, 2, 1)}.

## SECTION—C

9. Attempt all of the following questions : 8×2=16

(i) If $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, find the rank of $A^2$.



(ii) Define consistency of a system of linear equations.

(iii) Examine the linear independent or dependent of the rows of the matrix :

$\begin{bmatrix} 3 & 2 & 4 \\ 1 & 0 & 2 \\ 1 & -1 & -1 \end{bmatrix}$, hene find its rank.

(iv) Prove that the eigenvalues of an idempotent matrix are either zero or unity.

(v) Under what condition on the scalar $b \in R$, are the vectors $(b, 1, 0), (1, b, 1)$ and $(0, 1, b)$ in $V_3(R)$ linearly dependent?

(vi) Discuss whether or not $R^2$ is a subspace of $R^3$. Justify.

(vii) Show that the mapping $T : R^2 \to R$ defined by $T(x, y) = |2x - 3y|$ is not a linear transformation.

(viii) Verify Rank-Nullity theorem for the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$.