Roll No. .... Total Pages : 4

13602/NH

A/2111

DIFFERENTIAL EQUATIONS

Paper–II

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt **one** question each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION—A

1. (a) Solve the differential equation :

$$\left(1+y^2\right)dx+\left(1+x^2\right)dy=0.$$ 3

(b) Solve the differential equation :

$$\left(x^2+y^2+2x\right)dx+2y\,dy=0.$$ 3

2. Solve the differential equation :

$$\frac{dy}{dx}=\frac{x-y+2}{x+y-1}.$$ 6

3. Solve the differential equation :

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2+e^x+\cos 2x.$$ 6

4. (a) Solve $\left(D^4-2D^2+D^2\right)y=x^2.$ 3

(b) Explain the method of variation of parameters with example. 3

SECTION—B

5. Use operator method to find the general solution of the linear operator :

$$2\frac{dx}{dt}+\frac{dy}{dt}-x-y=1$$

$$\frac{dx}{dt}+\frac{dy}{dt}+2x-y=t.$$ 6

6. Solve Legendre's equation :

$$\left(1+x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+m\left(m+1\right)y=0,$$ m is any real or complex in power series about 0. 6

7. State and prove Rodrigue's formula. 6

8. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx=0;$ if $m \neq n.$ 6

SECTION—E

9. Attempt all the following questions : 8×2=16

(i) Define order and degree of a differential equation.

(ii) Solve the differential equation :

$$\sec^2 x \tan y \, dx + \tan x \sec^2 y \, dy = 0.$$

(iii) Find I.F. by inspection and solve :

$$x\,dx + y\,dy + 4y^3\left(x^2+y^2\right)dy = 0.$$

(iv) Solve : $\left(D^4-m^4\right)y=0.$

(v) Define Bessel's equation and function.

(vi) Show that $\int_{-1}^{1} P_2(x)dx=0.$

(vii) Define the Frobenius method.

(viii) Prove that $J_n(-x)=(-1)^n J_n(x),$ if n is positive integer.