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Roll No. .... Total Pages : 4

13484/NJ

E–27/2111

LINEAR INTEGRAL EQUATIONS

Paper–505

Semester–V

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. Convert the following $I \vee P$ into Volterra Integral Equations : $y''(x) + x\, y(x) = 1$, $y(o) = y'(o) = 0$. 10

2. Explain the method to find the solution of Volterra Integral Equation of second kind by Successive Approximation. 10

3. Solve by the method of Successive Approximations :

$$y(x) = e^{x^2} + \int_0^x e^{x^2 - t^2}\, dy(t)dt.$$ 10

4. Solve : $y(x) = \cos x + \lambda \int_0^{\pi} \sin x\, y(t)dt.$ 10

SECTION—B

5. State and prove first Fundamental theorem. 10

6. Determine $D(\lambda)$ and $D(x, t : \lambda)$ and hence solve the integral equation : 10

$$y(x) = e^x + \lambda \int_0^1 2\, e^x e^t\, y(t)dt.$$

7. State and prove Hadamard theorem. 10

8. Using Fredholm determinants, find the resolvent Kernel $K(x, t) = xe^t$, $a = 0$, $b = 1$. 10

SECTION—C

9. Write short notes on the following : 10×3=30

(i) Define the Singular Integral Equations and given example.

(ii) Define Reciprocal functions.

(iii) Define the Abel's problem.

(iv) Define the Volterra Integral Equation.

(v) Show that $y(x) = 1 - x$ is a solution of :

$$\int_0^x e^{x-t} y(t)dt = x.$$

(vi) State Schwarz's inequality.

(vii) State Dirichlet problem.

(viii) Define the Symmetric kernel.

(ix) Prove that eigen value of Symmetric kernel are real.

(x) Using Fredholm determinants, find the resolvent kernel of :

$$K(x, t) = 2x - t, \quad \begin{array}{l} 0 \le x \le 1 \\ 0 \le t \le 1 \end{array}$$