

**E-26/2111**  
STATISTICAL METHODS-I-306  
(Semester-III)

Time : Three Hours]

[Maximum Marks : 70

**Note** : Attempt *five* questions in all, selecting *two* questions each from Section A and B carrying 10 marks each and the compulsory Section C consisting of ten short answer type questions having 3 marks each. Use of non-programmable scientific calculator is allowed.

**SECTION-A**

I. If X and Y are independent continuous random variables, then derive the distribution of X - Y. (10)

II. Discuss the concept of distribution of a sample. Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal population with mean  $\mu$  and variance  $\sigma^2$ . Then show that

$$\frac{ns^2}{\sigma^2} = \sum_{i=1}^n \left( \frac{X_i - \bar{X}}{\sigma} \right)^2$$
 is a  $\chi^2$ -variate with  $n$  degrees of freedom. (10)

III. Discuss the standard error of a statistic. Compute the standard error of the sample proportion, the sample mean and the sample variance. (10)

IV. For a random sample  $X_1, X_2, \dots, X_n$  from  $N\left(\mu, \frac{\sigma^2}{n}\right)$ ,

find the mean and variance of  $S = \left[ \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} \right]^{\frac{1}{2}}$ .

(10)

### SECTION-B

- V. (a) A random sample of size 16 has 53 as mean. The sum of squares of deviations from mean is 150. Can this sample be regarded as taken from the population having 56 as mean?
- (b) Explain the steps in the testing of the mean of the univariate normal distribution. (6+4=10)
- VI. (a) Discuss the test of significance for the sample correlation coefficient in sampling from a bivariate normal distribution.
- (b) A coefficient of correlation of 0.2 is derived from a random sample of 625 pairs of observations. Is this value significant? What are the 95% and 99% confidence limits for the correlation coefficient in the population? (4+6=10)

- VII. (a) A random sample of 20 observations gave a standard deviation of 3.72. Is this compatible with the hypothesis that the sample is taken from a normal population with variance 4.35 ?
- (b) Explain the confidence limits for the population mean  $\mu$  for the critical values of the  $t$ -distribution.  
(5+5=10)
- VIII. (a) For 22 rivets produced by a firm A, the standard deviation of the diameters is 2.9 mm while for 16 rivets produced by firm B, the standard is 3.8 mm. Compute the statistic that will be used to test whether the products of firm A have same variability as those of firm B.
- (b) Explain the paired  $t$ -test for difference of means.  
(6+4=10)

**SECTION-C**  
**(Compulsory Question)**

- IX. (a) If the cumulative distribution function of the random variable  $X$  is  $F(x)$ , what is the distribution of  $aX$  for a constant  $a$  ?
- (b) Find the standard error of sample mean.
- (c) Define a random sample, population parameter and sample statistic.
- (d) Define the two types of errors in testing of hypothesis.
- (e) Define the  $p$ -value of a test statistic for a right tailed test and for a left tailed test with diagram.

- (f) Define the test statistic, null hypothesis and the conclusion in testing of variance of the univariate normal distribution.
  - (g) Define the F-statistic.
  - (h) A sample of size 9 from a normal population has mean 15.8 and variance 10.3. Find a 99% confidence interval for population mean.
  - (i) Give the statement of the Chi-square statistic.
  - (j) In a sample of 8 observations, the sum of squared deviations of items from their mean was 94.5. In another sample of 10 observations, the value was found to be 101.7. Test whether the difference is significant at 5% level. (10×3=30)
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