Roll No. .... Total Pages : 4

13481/NJ

E–26/2111

MATHEMATICAL METHODS

Paper–303

Semester–III

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. State and Prove Rodrigue's formula for Legendre Polynomials. 10

2. Show that $J_n(x)$ is the coefficient of z^n in the expansion of $\exp\left(\frac{x}{2}\left(z-z^{-1}\right)\right)$ in ascending and descending powers of z, where n is a positive integer. 10

3. Discuss the Orthogonality property of Chebyshev Polynomials. 10

4. Write Chebyshev's equation. Hence show that Chebyshev's polynomials of first and second kind are independent solutions of Chebyshev's equation. 10

SECTION—B

5. Let $L^{-1}(F(s)) = f(t)$ and $L^{-1}(G(s)) = g(t)$, then $L^{-1}(F(s)\cdot G(s)) = \int_0^t f(s)g(t-s)\,ds$. 10

6. Solve the differential equation $y'' - 4y' + 4y = 0$ using Laplace transform, where $y(0) = 0$ and $y'(0) = 3$. 10

7. Let f(t) be a periodic function with period k. Then the Laplace transform of f(t) is:

$$L(f(t)) = \frac{1}{1-e^{-sk}}\int_0^{\infty} e^{-st} f(t)\,dt.$$ 10

8. State and prove Parseval's identify for Fourier transforms. Hence show that :

$$\int_0^{\infty} \frac{dx}{\left(1+x^2\right)^2} = \frac{\pi}{4}.$$ 10

SECTION—C

9. Write answer the following : 10×3=30

(i) Write the generating function of Legendre polynomials.

(ii) Show that $\int_{-1}^{1} P_m(x) P_n(x)\,dx = 0 \quad m \neq n.$

(iii) Show that $T_{m+1}(x) - 2xT_m(x) + T_{m-1}(x) = 0.$

(iv) Show that $P_0(x) = 1$ and $P_n(-x) = (-1)^n P_n(x).$

(v) Write the generating function of chebyshev polynomials.

(vi) Find the Laplace transform of sin(at) and cos(at), where a > 0.

(vii) Prove that

$$L\{c_1 f(x) + c_2 g(x)\} = c_1 L\{f(x)\} + c_2 L\{g(x)\},$$

where c_1 and c_2 are constants.

(viii) Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right).$

(ix) Define change of scale property for Fourier transform of f(x).

(x) Find the Laplace transform of $f(t) = \int_0^t \frac{\sin u}{u}\,du.$