

(vii) Find the equation of the cone with vertex at the origin and generators touching the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$.

(viii) Find the equation of an ellipse referred to any pair of conjugate diameters as axes.

(ix) Prove that the equation :

$$2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$$

represents a cone whose vertex is $\left(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6}\right)$.

(x) Prove that the locus of the foot of the perpendicular from a focus to a tangent of the conic $\frac{1}{r} = 1 - e \cos \theta$ is $r^2(1 - e^2) - 2e/r \cos \theta = l^2$. 10×3=30

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COORDINATE GEOMETRY—1102T

Semester—I

Time Allowed : Three Hours]

[Maximum Marks : 70

Note :- The candidates are required to attempt *five* questions in all, selecting *two* questions from each Section A and B and the compulsory question of Section C.

SECTION—A

1. Prove that the equation $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$ represents a parabola. Trace the curve and find the length of its latus rectum, the equation of the axis and the equation of the tangent at the vertex. 10
2. (i) Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the yz-plane.
 (ii) Find the length of perpendicular from the point (2, -1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. 5+5
3. (i) Prove that the locus of the middle points of focal chords of a conic section is a conic section.
 (ii) Find the equation of the asymptotes of the conic $\frac{1}{r} = 1 + e \cos \theta$. 5+5

4. (i) A circle of given radius passing through the focus F of a given conic intersects it in A, B, C, D. Show that FA.FB.FC.FD is constant.
- (ii) Reduce $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$ to the standard form and classify the conic. 5+5

SECTION—B

5. (i) Find the equation of right circular cone whose vertex is at $(1, -2, -1)$, semi-vertical angle 60° and the line $\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}$ as its axis.
- (ii) Show that :
 $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$
 represents a cone. Also find the coordinates of vertex of this cone. 5+5
6. (i) Find the equation of enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 1 = 0$ and having its generators parallel to the line $x = y = z$.
- (ii) The axis of a right circular cylinder of radius 2 has equations $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$. Find its equation. 5+5
7. (i) Find the equation of an ellipse referred to any pair of conjugate diameters as axes.
- (ii) Prove that the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ make equal angles with x-axis if $m_1 + m_2 + 2m_1m_2 \cos w = 0$. 5+5

8. (i) Prove that the general equation to a cone which touches the three coordinate planes is $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$.
- (ii) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 5+5

SECTION—C

9. (i) What is represented by the equation
 $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$?
- (ii) Find the angle between pair of lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and
 $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.
- (iii) Find the polar equation of a circle whose centre is $\left(3, \frac{\pi}{2}\right)$ and passing through $\left(4, \frac{\pi}{6}\right)$.
- (iv) Show that if the tangent at any point P of a conic meets the directrix in T, then angle TSP is right angle, where S is the focus of the conic.
- (v) Find the polar equations of the asymptotes of a conic.
- (vi) Find the equation of the conic with vertex at the origin and which passes through the curve given by
 $x^2 + y^2 + z^2 + x - 2y + 3z = 4$ and
 $x^2 + y^2 + z^2 + 2x - 3y + 4z = 5$.