Roll No. .... Total No. of Pages : 2

PC 13107-N

L-4/2111

DIFFERENTIABLE MANIFOLDS—MM-601/AMC-308

Semester—III

(Common for Math/AMC)

Time Allowed : Three Hours] [Maximum Marks : 70

Note :– Attempt *five* questions in all selecting *two* questions each from Section A and B carrying 10 marks each and the compulsory Section C consisting of *ten* short answer type questions having 3 marks each.

SECTION—A

1. Define the concept of tangent vector as an equivalence class of curves. Show that the tangent space is a vector space over reals.

2. Discuss the concept of alternating covariant and contravariant tensors or order 'r' with reference to the permutation group of naturals thus leading to the symmetrizing and an alternating map. Also prove any two properties of the wedge product of two skew symmetric covariant tensors of order two.

3. Is the difference tensor of two connections skew symmetric ? Prove that two connections D and $\bar{D}$ are equal iff they have same geodesics and same torsion tensors.

4. Define the Lie-derivative of a tensor product space. Prove that $L_X = C_X \circ d + d \circ C_X$ where C_X is the Contraction map, L is the Lie derivative and d is the exterior derivative. 2×10=20

SECTION—B

5. Explain the local coordinate approach to the Torsion tensor. Prove that an affine connection can be decomposed into a sum of a multiple of its torsion tensor and a torsion free connection.

6. State and prove Bianchi's first identity. State the case when the connection is symmetric.

7. Prove that in the Koszul formula in a semi-Riemannian manifold, the connection defined is linear and metric compatible.

8. State and prove Schur's theorem. 2×10=20

SECTION—C

9. (a) Explain the basis of a wedge product space.

   (b) For the real valued functions f, g, h on $\mathfrak{R}^2$, evaluate $df \wedge dg \wedge dh$.

   (c) Define a Contraction map. Prove one property of the same.

   (d) Differentiate between a tensor product and a wedge product.

   (e) State and prove any two properties of Riemannian curvature tensor.

   (f) Prove any two properties of Christoffel symbols.

   (g) State and prove any two properties of the Lie-bracket.

   (h) Discuss the Gauss and Weingarten formulae in the theory of submanifolds.

   (i) Compute the exterior product $(6du^1 \wedge du^2 + 27du^1 \wedge du^3) \wedge (du^1 + du^2 + du^3)$.

   (j) Prove that Jacobain preserves Lie-bracket. 10×3=30