Roll No. .... **Total No. of Pages : 2**

PC 12979-N

K-10/2111

TOPOLOGY—MAT-1103T/AMCM-1103T

Semester—I

(Common for Math/AMC)

Time Allowed : Three Hours] [Maximum Marks : 70

Note :– The candidates are required to attempt *two* questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of *ten* short answer type questions carrying 3 marks each.

SECTION—A

1. How many sequences of terms consisting of only 0 or 1 are there ? Justify your answer. Prove that there cannot be a surjection from any set X to its set of all subsets **P**(X).

2. What are equivalent bases for a topological space ? Give an example of equivalent bases. Prove a necessary and sufficient condition for two bases to be equivalent.

3. Prove that a map $f : X \to Y$ is a continuous open bijection if and only if it has a continuous inverse.

4. Let $\{A_\alpha\}_\alpha$ be a neighborhood-finite family of subsets of a space X. Prove that union of the closures of all the A_α is a closed set.

Further, prove that if all the A_α are also closed and cover the whole space X, then a subset B of X is closed in X if and only if each $B \cap A_\alpha$ is closed in the subspace A_α.

SECTION—B

5. Prove that the product of connected spaces is connected.

6. Prove that an open subset of the Euclidean space $\mathbf{R}^n$ is connected if and only if it is path connected.

7. Prove that every separable metric space second countable.

8. What is a totally bounded metric space ? Prove that every sequentially compact metric space is totally bounded.

SECTION—C

9. (i) Give two bases for a discrete topology on the real line.

   (ii) Is Sierpinski space metrizable ? Justify.

   (iii) What is the frontier of the set of all rational numbers in the real line with the usual topology ? Justify your answer.

   (iv) What do you understand from the statement that subspace of a subspace is a subspace ?

   (v) Infinite product of proper open subsets of a space is Never open. Why ?

   (vi) Is the product of totally disconnected space always totally disconnected ? Justify.

   (vii) Give an example of a connected space which is not path connected.

   (viii) Prove that in Hausdorff spaces sequences have at most one limit.

   (ix) Define one point compactification of a space.

   (x) What is a locally compact space ? Give an example of a locally compact space which is not compact.