PC-12978/N

K-10/2111

MATHEMATICAL ANALYSIS-I

(Math 1102 T/AMCM-1102 T)

(Semester–I)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *two* questions each from Section A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION–A

I. State and prove implicit function theorem.

II. If m is countably additive, translation invariant measure defined on a σ algebra containing P then P is not measurable.

III. Show that the family of measurable sets is an algebra of sets.

IV. A linear operator T on a finite dimensional vector space X is one to one if and only if R(T) = X, where R(T) is range of T.

SECTION–B

V. State and prove Vitali's lemma.

VI. Let f be defined and bounded on a measurable set E with mE is finite. In order that $\inf_{\psi \geq f} \int_E \psi = \sup_{f \geq \varphi} \int_E \varphi$.

VII. A function f is of bounded variation on $[a, b]$ iff it can be written as a difference of two monotone real valued functions on $[a, b]$.

VIII. Let f be an increasing real valued function defined on $[a, b]$. Then f is differentiable almost everywhere. Moreover, the derivative f' is measurable and

$$\int_a^b f'(x)dx \leq f(b) - f(a).$$

SECTION–C

IX. (a) Define outer measure of a set. If $m^*E = 0$, then E is measurable.

(b) Suppose f maps convex set $E \subset R^n$ into R^m, f is differentiable on E and there exists a real number M such that $\|f(x)\| \leq M$, $\forall\, x \in E$, then

$$|f(b) - f(a)| \leq M\,|b - a|, \forall\, a, b \in E.$$

(c) Show that outer measure of a countable set is zero.

(d) Let $\{E_i\}$ be a sequence of measurable functions such that E_i's are pairwise disjoint then

$$m\bigcup_{i=1}^{n} E_i = \sum_{i=1}^{n} mE_i .$$

(e) Let T_1 and T_2 be two linear operators s.t. $T_1, T_2 \in L(R^n)$, then $\|T_1 + T_2\| \leq \|T_1\| + \|T_1\|$.

(f) If f is a integrable function defined on E, then $|f|$ is also integrable and $|\int f| \leq \int |f|$.

(g) Define simple function. Show that product of two simple functions is also simple.

(h) Let $<f_n>$ be a sequence of measurable functions that converges in measure to f, then there is a subsequence $<f_m>$ that converges to f almost everywhere.

(i) If f is absolutely continuous on $[a, b]$ then f has a derivative almost everywhere on $[a, b]$.

(j) State and prove Jensen inequality.