Total Pages : 3

PC-12977/N

K-10/2111

ALGEBRA-I (Math 1101 T/AMCM-1101 T)

(Semester–I)

(Common for AMC and Math)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *five* questions in all, selecting *two* questions from each Section A and B carrying 10 marks each and compulsory question of Section C carrying 30 marks.

SECTION–A

I. Prove that a group G is solvable iff it has a normal series with abelian factors. Deduce that a finite group is solvable iff its composition factors are cyclic groups of prime orders. (10)

II. Prove that subgroup and homomorphic image of a nilpotent group is nilpotent. Is the converse true? Prove or disprove. (10)

III. Let H be a normal subgroup of alternating group A_n, $n \geq 5$. Prove that H contains all the 3-cycles of S_n. (10)

IV. State and prove Burnside Theorem. (10)

SECTION–B

V. Let G be a group of order pq, where p and q are prime numbers such that $p < q$ and $p \dagger (q - 1)$. Prove that G is cyclic. What will happen if $p = q$? (10)

VI. Prove that two cyclic groups of same order are isomorphic. (10)

VII. Prove that in a non-zero commutative ring R with unity, an ideal M is maximal iff R/M is field. (10)

VIII. For a ring R with unity, find all the ideals of the matrix ring $M_n(R)$. (10)

SECTION–C
(Compulsory Question)

IX. (a) Prove that Jordan-Holder theorem implies the fundamental theorem of arithmetic.

(b) Define normal and subnormal series of a group. Give an example of a subnormal series which is not normal.

(c) Prove that any *two* conjugate permutations in S_n have the same cyclic structure.

(d) Prove that the center of a group is its normal subgroup.

(e) Let $A = \{1, 2, 3\}$ and $G = S_3$. Let $*: G \times A \to A$ s.t., $\sigma * \alpha = \sigma(\alpha)$. Prove that $*$ is a group action and find all the stabilizers.

(f) State fundamental theorem of finitely generated abelian groups.

(g) Find all non-isomorphic abelian groups of order 720.

(h) Prove that there is no simple group of order 56.

(i) Prove that intersection of two ideals of a ring is an ideal of the ring.

(j) Let G be a group of order p^n (p – prime and n – positive integer). Prove that Z(G) is non-trivial.

(10×3=30)