PC-11801/NJ

D-6/2111

ALGEBRA (GROUP AND RING THEORY)–501
Semester-V
(Common for MC & B.Sc. Hons. in Math)

Time : Three Hours] [Maximum Marks : 70

Note : Candidates are required to attempt *two* questions each from Section-A and B. Section-C will be compulsory. Each question of Section-A and B carries 10 marks and Section-C consists of 10 short answer type questions carry 3 marks each.

SECTION – A

I. State and Prove Zassenhaus Lemma.

II. State and Prove Schreier's Refinement theorem.

III. If I and J be any two ideals of a ring R, then Prove that I + J is an ideal of R and I + J = {I ∪ J}, is the smallest ideal of R containing I ∪ J.

IV. Let G be a group of order p^n, where p is a prime. Then show that G has a composition series such that all its composition factors are of order p.

SECTION – B

V. State and prove second Isomorphism theorem.

VI. Prove that every ring with unity can be embedded in a ring of endomorphisms of some additive abelian group.

VII. Show that every irreducible element in a principal ideal domain is a prime element.

VIII. Prove that every Euclidean domain is a Principal ideal domain.

SECTION – C

IX. (a) Define Homomorphism of Rings.

(b) Define Subring.

(c) Prove that the union of subrings need not be subring.

(d) What do you mean by ideal?

(e) Define Quotient Ring with an example.

(f) What do you mean by Prime element.

(g) Define Composition Series.

(h) Define soluable group.

(i) What do you mean by integral domain.

(j) Define maximal ideal. (10×3=30)
