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Roll No. .... Total Pages : 3

11772/NJ

D–3/2111

ABSTRACT ALGEBRA

Paper–352

Semester–V

Time Allowed : 3 Hours] [Maximum Marks : 45

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 7 short answer type questions carrying 3 marks each.

SECTION—A

1. If H and K are two subgroups of a group G then prove that HK is a subgroup of G if and only if HK = KH. 6

2. Prove that the order of each subgroup of a finite group is a divisor of the order of the group. 6

3. Prove that every subgroup of a cyclic group is cyclic. 6

4. Let N be a a normal subgroup of a group G. Show that G/N is abelian if and only if for all $x, y \in G, xyx^{-1}y^{-1} \in N$. 6

SECTION—B

5. If I and J be any two ideals of a ring R, then show that IJ is an ideal of R. 6

6. Show that in the ring Z of integers, an ideal (n) is maximal if and only if n is prime, where $(n) = \{nx : x \in Z\}$. 6

7. Let I and J be two ideals of a ring R then show that $I/(I \cap J) \simeq (I + J)/J$. 6

8. Prove that every Euclidean domain is principal ideal domain. 6

SECTION—C

9. Answer the following : $7 \times 3 = 21$

(i) Define group, subgroup and normal subgroup.

(ii) If H and K be any two subsets of a group G. Then show that $(HK)^{-1} = K^{-1} H^{-1}$.

(iii) How many elements of order 5 are there in A_6?

(iv) What is class equation? Give one application of it?

(v) Let R be any field then show that it is also an integral domain.

(vi) Check whether the mapping $f : Z|\sqrt{2}| \to Z|\sqrt{3}|$ defined by $f\left(a+b\sqrt{2}\right) = a + b\sqrt{3}$ is a ring homomorphism or not?

(vii) Define the Euclidean domain and give its properties.