Roll No. .... Total Pages : 6

11754/NJ

D–1/2111

PROBABILITY (Theory)–I

Paper–1104T

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 30

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 4 marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION—A

1. (a) What is meant by Mutually Exclusive events ? Give example of (i) Three mutually exclusive events, (ii) Four events which are not equally likely.

(b) Find the probability that among three random digits these occur 0, 1, 2 repetitions.

2. (a) Give arcromatic definition of probability. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, find P(A) and P(B). Hence show that A and B are independent.

(b) What is the probability that the birthdays of r randomly selected persons will be all different ? $(r \leq 365)$.

3. (a) The events $B_1, B_2, \ldots. B_n$ are mutually exclusive and $B = \bigcup_{i=1}^{n} B_i$. Show that if $P(A / B_i) = P(B / B_i)$; i = 1, 2, ..., n then $P(A / B) = P(B / B)$.

(b) What is the probability that two throws with three dice each will show the same configuration (i) if the dice are distinguishable (ii) they are not.

4. (a) Let A and B be two arbitrary events. Show that $P(A \cup B) \le P(A) + P(B)$.

(b) A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelop just two consective letter TA are visible. What is the probability that the letter came from CALCUTTA ?

SECTION—B

5. (a) Explain what is meant by a random variable. Distinguish between a discrete and a continuous random variable. Also give example of each.

(b) Suppose that the random variable x has possible values 1, 2, 3, ... and $P(x = j) = \frac{1}{2^j}; j = 1, 2, 3, ...$ Compute P (x is even).

6. (a) Define expectation of random variable and write down its properties.

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

7. (a) Define moments and moment generating function of a random variable x. If M(t) is the moment generating function of a random variable x about the origin, show that the moment of order 'r' about origin (μ_r^1) is given by; $\mu_r^1 = \left[\frac{d^r M(t)}{dt^r}\right]_{t=0}$.

(b) If P(t) is the probability generating function for the random variable x, find the probability generating function for $\left(\frac{x-a}{b}\right)$.

8. (a) The joint probability density function of two dimensional random variable (x, y) is given by :

$$f(x, y) = \begin{cases} x^3y^3/16 & , \quad 0 \le x \le 2, 0 \le y \le 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the marginal density functions of x and y.

(b) The joint probability distribution of a pair of random variables (x, y) is given by :

y \ x	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Find marginal distributions and conditional distribution of x given y = 1.

SECTION—C

9. Answer the following questions :

(i) Explain the concepts of marginal and conditional distributions for the case of two dimensional random variables theoretically.

(ii) What do you mean by probability mass function and probability density function ? Explain.

(iii) What is the expectation of the number of failures preceeding the first success in an infinite series of independent trials with constant probability 'p' of success in each trial ?

(iv) Each of the 12 districts has 2 representatives. Find the probability that in a committee of 12 representatives chosen at random : (i) a given district is represented, (ii) all districts are represented.

(v) An Urn contains 6 red and 4 black balls. Two balls are drawn without replacement. Wht is the probability that the second ball is red if it is known that the first is red.

(vi) What are the objections raised in classical and statistical defentions of probability? If A and B are two mutually exclusive events and $P(A \cup B) \neq 0$, then also prove that

$$P(A / A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$

(vii) What are measures of dispersion ? How are measures used with respect to probability distribution ?