Roll No. .... Total No. of Pages : 3

PC 11478-NH

CS/2111
MATHEMATICAL METHODS–I, Paper–III, Opt. (i)
Semester—V

Time Allowed : Three Hours] [Maximum Marks : 40

Note :– The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

I. State and prove Riemann-Lebesgue Theorem of Fourier series. 6

II. (a) Obtain the Fourier series expansion of function

$f(x) = \sqrt{1 - \cos x}$ in $(-\pi, \pi)$. 3

(b) Express the following function as half range cosine series

$$f(x) = \begin{cases} x; & 0 < x < \frac{\pi}{2} \\ \pi - x; & \frac{\pi}{2} < x < \pi \end{cases}$$ 3

III. (a) Find the complex form of the Fourier series of the function

$$f(x) = \begin{cases} 2; & -2 \le x \le 1 \\ 0; & 1 \le x \le 2 \end{cases}$$ 3

(b) Find the Fourier series of the function

$$f(x)=\begin{cases}-\cos x; & -\pi<x<0\\ \cos x; & 0<x<\pi\end{cases}$$ 3

IV. State Parceval's identity of Fourier series and use it for the function $f(x) = x^2$ in $(-\pi, \pi)$ to prove $\frac{\pi^4}{90} = 1+\frac{1}{2^4}+\frac{1}{3^4}+ \ldots \ldots \ldots \ldots$ 6

SECTION—B

V. State and prove second shifting property of Laplace transforms. Hence evaluate $L\{g(t)\}$ where

$$g(t) = \begin{cases} 0 & \text{if} \quad 0<t<\frac{1}{2}\\ t+\frac{3}{2} & \text{if} \quad t>\frac{1}{2}\end{cases}$$ 6

VI. (a) By use of Laplace Transforms, prove that $\int_0^\infty \sin x^2 dx = \sqrt{\frac{\pi}{8}}$. 3

(b) Evaluate $l\{\sin\sqrt{t}\}$ and hence evaluate $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$. 3

VII. State convolution theorem and use it to evaluate

$$L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}.$$ 6

VIII. State Heaviside's Expansion formula and use it to evaluate

$$L^{-1}\left\{\frac{1}{(s^3-1)}\right\}.$$ 6

SECTION—C

IX. (a) Find the Fourier series of function $f(x) = x$ in $[-\pi, \pi]$.

(b) If f is a bounded and integrable function defined in $[-\pi, \pi]$, then prove that Fourier coefficients a_n and b_n of f tends to zero as $n \to \infty$.

(c) Define half range cosine series.

(d) State Dirichlet's conditions for Fourier expansion of functions.

(e) Evaluate $L^{-1}\left\{\frac{2s-1}{s^3-s}\right\}$.

(f) Evaluate $L^{-1}\left\{\frac{e^{-6s+1}}{(s-3)^5}\right\}$.

(g) Evaluate $L\left\{\int_0^t z^2 e^{-2z}\,dz\right\}$.

(h) Evaluate $L\{t^2 \sin at\}$. 8×2=16