PC 11476-NH

CS/2111
ALGEBRA—I
Paper—I
Semester—V

Time Allowed : Three Hours] [Maximum Marks : 40

Note :– The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

I. (a) Show that the set of all positive Rational Numbers form an infinite abelian group under composition defined as $a \quad b = \frac{ab}{3}$. 3

(b) If in a group G, $a^5 = e$ and $aba^{-1} = b^2$ for all $a, b \in G$. Prove that if $b \neq e$. Then prove that $O(b) = 31$. 3

II. Prove that a subgroup of a cyclic group is cyclic. 6

III. (a) If G and G' are two groups and f is an isomorphism from G into G', then

Prove that $O(a) = O(f(a))$ for all $a \in G$. 3

(b) Let H be a subgroup of a Group G, then prove that G is equal to the union of all right cosets of H in G. 3

IV. State and prove Fundamental Theorem of Group Homomorphism. 6

SECTION—B

V. Let I and J be two ideals of Ring R, then prove that I + J is the smallest ideal of R containing $I \cup J$. 6

VI. Let I and J be two ideals of a ring R. Then show that

$I/(I \cap J) \cong (I + J)/J$. 6

VII. Prove that every Euclidean domain i.e. E.D. is principal ideal domain i.e. P.I.D. 6

VIII. Prove that the set of rational numbers Q is a ring under the compositions

$a \odot b = a + b - 1$ and $a \oplus b = a + b - ab \ \forall \ a,b \in Q$. 6

SECTION—C

IX. (a) Let G be group of integers under addition and $G' = \{-1, 1\}$ be group under multiplication. Define a mapping $f : G \to G'$

as $f(x) = \begin{cases} 1 & \text{if n is even} \\ -1 & \text{if n is odd} \end{cases}$

Then prove that f is a Homomorphism.

(b) Let G be a group then prove that $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

(c) Prove that an infinite cyclic group has precisely two generators.

(d) If H is a subgroup of a group G such that $[G : H] = 2$, then prove that H is normal Subgroup of G.

(e) If R is a ring in which $a^2 = a$ for all $a \in R$, Then prove that R is a commutative Ring of characteristic 2.

(f) Let R and S be two rings. Then a Homomorphism $f : R \to S$ is one-one if and only if Ker $f = \{0\}$.

(g) Find Quotient Field of $Z[\sqrt{2}]$, where $Z[\sqrt{2}] = \{a + \sqrt{2}b ; a, b \in Z\}$.

(h) If $I = \{6n ; n \in Z\}$ be an ideal of ring Z of integers, write the composition Tables for quotient ring Z/I. 8×2=16