(g) If the greatest height attained by a projectile is $\frac{1}{4}$ of its range. Find the angle of projection.

(h) Two masses, m_1 and $m_2 (m_1 > m_2)$ are suspended by a light inextensible and flexible string passing over a smooth, fixed, light pulley. Find the acceleration of masses.   8×2=16

PC 11456-NH

BS/2111
MECHANICS—III
Semester—III

Time Allowed : 3 Hours]   [Maximum Marks : 40

Note :— The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

I. (a) The constituent forces of a couple of moment G acts at points A and B. If their lines of action are turned through a right angle, they form a couple of a moment H. When they both act at right angle to AB, show that they form a couple of moment $\sqrt{G^2 + H^2}$.   3

(b) Let two forces of magnitude P and Q act at such an angle that magnitude of resultant is P. Show that if P is doubled, the new resultant is at right angle to the force of magnitude Q and its magnitude will be $\sqrt{4P^2 - Q^2}$.   3

II. (a) Six coplanar forces act on a rigid body along the sides AB, BC, CD, DE, EF and FA of a regular hexagon ABCDEF of side 1 unit of magnitude 10, 20, 30, 40, P and Q units respectively. Find P and Q so that the system reduces to a couple. Also find the moment of the couple.   3

(b) Let G be the centroid of a triangle ABC, prove that forces acting at G and represented by GA, 2GB, 3GC have a resultant force represented by 3GH where H is the point on BC such that BH = 2HC.   3

III. Equal weights $\vec{W}$ and $\vec{W}$ are attached to two ends of a string passing over a smooth peg at Q. The two portions of the string are separated by a heavy beam AB of weight $\vec{W}'$, whose centre of gravity is at a distance a metres from A and b metres from B. Show that AB is inclined to the horizontal at an angle

$$\tan^{-1}\left[\frac{a-b}{a+b}\tan\left(\sin^{-1}\frac{W'}{2W}\right)\right].$$

6

IV. State and prove Varignon's theorem. 6

SECTION—B

V. (a) Two cars starts off the race with velocities u and v and travel in a straight line with uniform acceleration α and β; if the race ends in the dead heat, prove that the length of the course is $\dfrac{2(u-v)(u\beta - v\alpha)}{(\alpha - \beta)^2}$. 3

(b) A body projected vertically upwards from top of a tower reaches the ground in t_1 seconds. If it be projected vertically downwards from the same point with the same velocity, it reaches the ground in t_2 seconds. Prove that if simply let fall from the point, it will reach the ground in $\sqrt{t_1 t_2}$ seconds.

3

VI. A body moves down a smooth inclined plane under the action of gravity alone, discuss its motion. Also find the time to reach the highest point, distance of the highest point from the point of projection and time of flight. 6

VII. (a) A particle moves with an acceleration f given by f = –k v where v is the velocity of the particle and k is constant. Express :

(i) v in terms of t

(ii) x in terms of t

(iii) v in terms of x.

It is given that at t = 0, v = u and x = 0. 3

(b) A particle is executing S.H.M., A and B are the points at which its velocity is zero. It passes through a certain point P at intervals of 0.5 and 1.5 seconds with a speed of 3 m/sec. Determine the maximum speed and the ratio $\dfrac{AP}{PB}$.

3

VIII. A particle of mass m is projected from a fixed point with the velocity u in a direction making angle $\alpha\left(\neq\dfrac{\pi}{2}\right)$ with the horizontal. Neglecting air resistance. Find the time of flight, the horizontal range, the maximum horizontal range and the greatest height attained. 6

SECTION—C

IX. (a) Prove that the minimum magnitude of the resultant of concurrent forces P and Q is |P – Q|.

(b) Find two unlike parallel forces, acting at a distance of 10m from each other, which are equivalent to a force 100N acting at a distance 6m from the greater force.

(c) State Newton's laws of motion.

(d) What is the resultant of a force and a couple ?

(e) Find maximum and minimum velocities of a particle executing S.H.M.

(f) A stone falling under gravity travels 34.3 metres in the last second of its flight. Find the height through which it fell and the time of its falling.