(f) Let a function $T : R^2 \to R^2$ be defined as $T(x, y) = (x + 1, y)$. Is T a linear transformation ?

(g) Show that {(1, 2, 1), (0, 1, 1), (1, 0, 2)} is a basis of R^3 over R.

(h) If $B_1 = \{(1, 2), (0, 1)\}$ and $B_2 = \{(1, 1), (2, 3)\}$ are bases of R^2, find transition matrix from B_2 to B_1. 8×2=16

Roll No. .... **Total No. of Pages : 4**

PC 11436-NH

AS/2111
LINEAR ALGEBRA—III
Semester—I

Time Allowed : Three Hours] [Maximum Marks : 40

Note :– The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

I. State and prove Cayley-Hamilton theorem. 6

II. (a) Using Gauss Jordan Method, find the inverse of matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$ 3

(b) Show that the rank of the matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ is less than 3 if and only if either $a + b + c = 0$ or $a = b = c$. 3

III. (a) Is the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & -1 & 4 \end{bmatrix}$ diagonalizable ? 3

(b) If λ is an eigen value of an invertible matrix A over R, then show that λ^{-1} is an eigen value of A^{-1}. 3

IV. (a) Find the values of k so that the system of equations

$x - 2y + z = 0,\ 3x - y + 2z = 0,\ y + kz = 0$

has (i) a unique solution (ii) an infinite number of solutions. Also find solutions for these values of k. 3

(b) Find the value of k so that vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} k \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent. 3

SECTION—B

V. Let V be the set of all $n \times n$ symmetric matrices over field R. Let vector addition and scalar multiplication be defined as usual addition of matrices and multiplication of a scalar with a matrix. Show that V is a vector space over R. 6

VI. (a) If W_1 and W_2 are two subspaces of a vector space V over a field F, then $W_1 \cup W_2$ is subspace of V if and only if either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. 3

(b) Let V be a vector space over a field F. Then a finite subset $S = \{x_1, x_2, x_3, \ldots\ldots\ldots, x_n\}$ of non-zero elements of V is linearly dependent if and only if some element say $x_k (2 \le k \le n)$ of S can be written as a linear combination of its proceeding elements. 3

VII. Let $T : R^3 \to R^2$ be the linear transformation defined by

$T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$.

(i) Find the matrix of T w.r.t. following bases of R^3 and R^2 :

$B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and

$B_2 = \{(1, 3), (2, 5)\}$

(ii) Verify that $[T; B_1, B_2]\ [v; B_1] = [T(v); B_2]\ \forall\ v \in R^3$. 6

VIII. State Rank and Nullity theorem and verify this theorem for the linear operator :

$T : R^3 \to R^3$ be the linear transformation defined by

$T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. 6

SECTION—C

IX. (a) Define Row Rank of a Matrix.

(b) Show that at least one eigen value of every singular matrix is zero.

(c) Does the following system of equations have a non-zero solution ?

$x + y + z = 0,\ x + 2y + 3z = 0,\ x + 3y + 4z = 0$.

(d) Define Diagonalizable Matrix.

(e) Is the union of two subspaces of a vector space V, a subspace of V ? Justify.