

Roll No. ....

Total Pages : 4

**22021/NH**

**B/2111**

**ADVANCED CALCULUS**

Paper-I

Semester-III

Syllabus Dec.-20

Time Allowed : 3 Hours]

[Maximum Marks : 40

**Note :** The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

**SECTION—A**

1. State and prove Schwarz's theorem. 6

2. State and prove Taylor's theorem for functions of two variables. 6

3. If  $z = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ , prove that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \tan$

and  $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = \tan^3 z.$  6

4. A rectangular box, open at the top, is to have volume of 32 Cubic metres. Find the dimensions of the box requiring least material for its construction. 6

**SECTION—B**

5. Evaluate  $\iint_A xy \, dx \, dy$ , where A is the region common to the circles  $x^2 + y^2 = x$ ,  $x^2 + y^2 = y.$  6

6. Evaluate by changing the order of integration

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 \, dy \, dx}{\sqrt{y^4 - a^2 x^2}}.$$
 6

7. Evaluate  $\iint \sqrt{\frac{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}}} dx dy$  over the positive

quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . 6

8. Find the area enclosed by the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ ,  $a > 0$ . 6

### SECTION—C

9. Answer the following : 8×2=16

(i) Prove that  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0$ .

(ii) Given  $u = e^x \cos y + e^y \sin z$ . Find all first and second order partial derivatives.

(iii) What do you mean by Homogeneous functions ?

(iv) State Euler's theorem on Homogeneous functions.

(v) Define Jacobian of a function.

(vi) Define Limit of a function of two variables.

(vii) Verify that :

$$\int_1^2 \left( \int_3^4 (xy + e^y) dx \right) dy = \int_3^4 \left( \int_1^2 (xy + e^y) dy \right) dx.$$

(viii) Let  $f(x, y) = \sqrt{x^4 + y^4 + 1}$ . Evaluate  $f_x(1, 2)$  and  $f_y(1, 2)$ .