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Roll No. .... Total Pages : 4

13601/NH

A–2111

CALCULUS–I

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION—A

1. (a) Examine the continuity of the function : 3

$$f(x) = [x] + [1 - x] \text{ at } x = 0.$$

(b) If $y = \sin(m \sin^{-1} x)$, prove that : 3

$$(1 - x^2) y_2 - x y_1 + m^2 y = 0.$$

2. Prove that : 6

$$\frac{d^n}{dx^n}\left[\frac{\log x}{x}\right] = \frac{(-1)^n n!}{x^{n+1}}\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \dots - \frac{1}{n}\right].$$

3. (a) Find the points of inflection of the curve $y = \dfrac{x^2 + 1}{x^2 - 1}$. Also determine the values of x for which function is concave upward and concave downward. 3

(b) Find the asymptotes of the curve : 3

$$x^3 + y^3 - 3axy = 0.$$

4. Trace the curve $y = (x+1)^2 (x-3)$. 6

SECTION—B

5. Prove that the function $f(x, y) = \sqrt{|xy|}$ is not differentiable at the origin, but that f (x, y) is

continuous at the origin and both f_x, f_y exist at the origin and have the value zero. 6

6. If $z(x+y) = x^2 + y^2$, then $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. 6

7. (a) Expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) upto the terms of the second degree. 3

(b) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. 3

8. (a) Let $f : R^3 \to R$ be defined as $f(x, y, z) = xyz$. Determine x, y, z for maximum of f where $xy + 2yz + 2zx = 108$. 4

(b) Verify Euler's theorem for the function :

$z = x^4 \log \frac{y}{x}$. 2

SECTION—C

9. Answer all the following questions : 8×2=16

(i) Find the nth derivative of $\sqrt{zx+b}$.

(ii) State Leibnitz's theorem on derivatives.

(iii) Define the Oblique asymptote.

(iv) Determine a and b so that curve $y = ax^3 + bx^2$ has a point of inflection at (–1, 2).

(v) If $u = e^x \sin y$, $x = \log t$, $y = t^2$, find $\frac{du}{dt}$ by partial differentiation. Also verify with direct calculations.

(vi) Show that : $f(x, y) = x^2 + y - 1$ is continuous at (1, –2).

(vii) Find points of extreme values, if any of the function $f(x, y) = x^3 + 3x + y^3 - y + 4$.

(viii) Find the percentage error in calculating the area of a rectangle when an error of 2 percent is made in measuring its sides.