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PC 11435-NH

AS/2111
DIFFERENTIAL EQUATIONS—II
Semester—I

Time Allowed : Three Hours] [Maximum Marks : 40

Note :– The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

I. (a) Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$. 3

(b) Solve the differential equation $\frac{dy}{dx} + y \cos x = y^2 \sin 2x$. 3

II. Solve $(D^2 - 1)y = x^2 \cos x$. 6

III. (a) Solve by method of variation of parameters the differential equation $\frac{d^2y}{dx^2} + 9y = \sin 3x$. 3

(b) Solve the following differential equation :

$$\frac{dy}{dx} = \frac{x + 2y + 1}{2x + 4y + 3}.$$ 3

IV. (a) Show that the following functions are linearly independent yet their Wronskian vanishes on the given interval :

$$f_1 = \begin{cases} x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}, \quad f_2 = \begin{cases} 0, & x \geq 0 \\ x^2, & x < 0 \end{cases}$$ 3

(b) Prove that $\frac{1}{D-a} V = e^{ax} \int V e^{-ax} \, dx$, no arbitrary constant being added. 3

SECTION—B

V. (a) Solve the System by Using Operator Method :

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t \text{ and } 2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2.$$ 3

(b) Solve the following differential equation :

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65 \sin(\log x).$$ 3

VI. Solve in series the Bessel's equation of zero order :

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0.$$ 6

VII. Prove that if $\int_{-1}^{1} P_m(x) P_n(x) \, dx = 0$ if $m \neq n$. 6

VIII. (a) Prove that :

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x)\right].$$ 3

(b) For integral values of n, show that $J_{-n}(x) = (-1)^n J_n(x)$. 3

SECTION—C

IX. (a) Show by Wronskian that the following functions are linearly independent

x, x^3, x^4 are linearly independent if $x \neq 0$.

(b) Find the order and degree of the differential equation

$$y = x\frac{dy}{dx} + a\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{2}}.$$

(c) Solve the differential equation $D^2y = e^x \cos x$.

(d) Define exact differential equation by giving one example.

(e) Show that $P_n(1) = 1$.

(f) Show that $y = x^n J_n(x)$ is a solution of

$$x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0.$$

(g) Solve the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x$.

(h) State Rodrigues' Formula. 8×2=16