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PC 11434-NH

AS/2111
CALCULUS—I
Semester—I

Time Allowed : Three Hours] [Maximum Marks : 40

Note :– The candidates are required to attempt *two* questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of *eight* short answer type questions carrying 2 marks each.

SECTION—A

I. (a) Show that the function $(x)=\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$, where $x \neq 0$ and $f(0) = 0$ is discontinuous at $x = 0$. 3

(b) If $(x) = ax^3 + 3bx^2$, determine a and b so that the graph of f has a point of inflexion at (–1, 2). 3

II. State Leibnitz Theorem and Prove that

$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0$, if $y=\log(x+\sqrt{x^2+1})$.

Also find $y_n(0)$. 6

III. (a) Find the equation of the hyperbola having $x + y - 1 = 0$ and $x - y + 2 = 0$ as its asymptotes and passing through origin. 3

(b) Examine the following curve for concavity, convexity and points of inflexion

$y = x^3 + 3x^2 - 24x + 20.$ 3

IV. Trace curve $y = x^3$. 6

SECTION—B

V. (a) Discuss the continuity of the following function at (0, 0)

$$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$ 4

(b) If $zk = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Using Euler's theorem on homogenous function, prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z.$$ 2

VI. If f(x, y) is a real valued function with domain $Df \subseteq R^2$ s.t. f_y exists in a neighbourhood of $(a, b) \in Df$ and if f_{xy} is continuous at (a, b) then $f_{yx}(a,b)$ exists and $f_{xy}(a, b) = f_{yx}(a, b)$. 6

VII. (a) Obtain Taylor's expansion for $f(x, y) = e^{xy}$ at (1, 1) upto third term. 3

(b) By Using Definition, Prove that $\lim\limits_{(x, y)\to(1, 1)}(x^2 + 2y) = 3.$

3

VIII. Find the Maximum and Minimum value of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$. 6

SECTION—C

IX. (a) Using definition of limit, prove that $\lim\limits_{x\to 2}(4x - 5) = 3.$

(b) If $y = \sin^{-1} x$, show that $(1-x^2)\dfrac{dy^2}{dx^2} - x\dfrac{dy}{dx} = 0.$

(c) Find point of inflexion on the graph for function $y = x^4$.

(d) State Euler's theorem in homogenous function of two variables.

(e) Let $f : R^2 \to R$ be defined by

$$f(x, y) = \begin{cases} 1, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

show that the first order partial derivatives of f at (0, 0) do not exist.

(f) Show that the asymptotes of the curve $x^2y^2 = a(x^2 + y^2)$ form a square of side 2a.

(g) State Young's Theorem.

(h) Show that $\lim\limits_{x\to 1}\dfrac{1}{x-1}$ does not exist. 8×2=16