

Statics-III

Sem-III (Syll-Dec-2020)

Time :- 3hrs

M.M. - 40

SECTION-A

 $2 \times 6 = 12$

- I (a) The resultant of two forces \vec{P} and \vec{Q} is at right angle to \vec{P} . The resultant of \vec{P} and \vec{Q}' acting at same angle is at right angle to \vec{Q}' . Show that $P^2 = QQ'$. (3)
- I (b) Prove that the algebraic sum of the moments of the forces forming a couple about any point in their plane is constant. (3)
- II State and prove Varignon's theorem. (6)
- III (a) If forces of magnitudes P, Q and R act at a point parallel to the sides BC, CA and AB respectively of a triangle ABC, prove that the magnitude of their resultant force is equal to $\sqrt{P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C}$ (3)
- III (b) The constituent forces of a couple of moment G acts at points A and B. If their lines of action are turned through a right angle, they form a couple of a moment H. When they both act at right angle to AB, show that they form a couple of moment $\sqrt{G^2 + H^2}$. (3)
- IV (a) Prove that algebraic sum of resolved parts of any two concurrent forces along a given direction in their plane is equal to the resolved part of their resultant along the same direction. (3)
- IV (b) Six coplanar forces act on a rigid body along the sides AB, BC, CD, DE, EF and FA of a regular hexagon ABCDEF of side 1 unit of magnitude 10, 20, 30, 40, P and Q units respectively. Find P and Q so that the system reduces to a couple. Also find the moment of the couple. (3)

SECTION-B

 $2 \times 6 = 12$

- V (a) Prove that the centre of gravity of a solid hemisphere lies at a point on the central radius that divides it in the ratio 3:5 from the centre. (3)
- V (b) A solid hemisphere and a solid right circular cone have their bases joined together, the bases being of the same size. Find the semi-vertical angle of the cone so that centre of gravity of the combined body may be at the centre of the common base; two solids being made of the same material. (3)
- VI Two weights P and Q are suspended from a fixed point O by strings OA and OB which are kept apart by a light rod AB. If the strings make angles α and β with the rod, show by Stating

the Lami's theorem, that the angle θ which the rod makes with the vertical is given by

$$\tan \theta = \frac{P+Q}{P \cot \alpha - Q \cot \beta} \quad (6)$$

VII (a) A uniform ladder rests with one end on a horizontal floor and other against a vertical Wall, the coefficients of friction being $\frac{3}{7}$ and $\frac{1}{3}$ respectively. Find the inclination of the Ladder when it is about to slip. (3)

VII (b) A uniform rod rests with one extremity against a rough vertical wall, the other end being supported by a string of equal length fastened to a point in the wall. Prove that least angle which the string can make with the wall is $\tan^{-1} \left(\frac{3}{\mu} \right)$, where μ is the coefficient of friction. (3)

VIII Equal weights \vec{W} and \vec{W} are attached to two ends of a string passing over a smooth peg at O. The two portions of the string are separated by a heavy beam AB of weight \vec{W}' , whose centre of gravity is at a distance a from A and b from B. Show that AB is inclined to the horizontal at an angle $\tan^{-1} \left[\frac{a-b}{a+b} \tan \left(\sin^{-1} \frac{W'}{2W} \right) \right]$. (6)

SECTION-C

IX (a) State Parallelogram law forces.

IX (b) State the conditions of equilibrium for any number coplanar concurrent forces.

IX (c) Let four forces of magnitudes P, P, P and $2P$ acts along sides DA, AB, BC and CD of a square ABCD. Find a point on AD such that the algebraic sum of moments of forces about it vanishes.

IX (d) The resolved part of a force of magnitude 32N in a direction is 16N. Find its inclination with the force and other resolved part.

IX (e) Give laws of limiting friction.

IX (f) Find the centre of gravity of uniform rod.

IX (g) State λ - μ theorem.

IX (h) What is the resultant of a force and a couple?

21086 / NH (2 × 8 = 16)