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Roll No. .... Total Pages : 4

13721/NH

C/2111

ALGEBRA

Paper–I

Semester–V

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION—A

1. (a) If H and K are two subgroups of a group G,

then show that HK is a subgroup of G iff HF = KH.

(b) Let G be a finite group and let $a \in G$ be an element of order n. Then show that $a^m = e$ if and only if n is a divisor of m. 6

2. (a) A subgroup H of group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(b) Prove that any infinite group cyclic group is isomorphic to the additive group of integers. 6

3. Prove that any finite semi-group is a group iff both the cancellation laws hold. 6

4. (a) State and prove first theorem on group homomorphism.

(b) Prove that there are only two groups of order. 6

SECTION—B

5. Prove that the characteristic of an integral domain is either prime or zero. In particular, a characteristic of a filed of either prime or zero. 6

6. If I and J be any two ideals of a ring R, then prove that IJ is an ideal of R. Moreover $IJ \subseteq I \cap J$. 6

7. Prove that a commutative ring R with identity is simple if and only if R is a field. 6

8. Prove that in an integral domain every prime element is an irreducible element. Also show that the converse may not be true. 6

SECTION—C

9. Answer the following questions : 8×2=16

   (i) In a semi group show that cancellation law may not hold.

   (ii) State First theorem on group homomorphism.

   (iii) Give an example of a non-sbelian group in which all the subgroups are normal.

   (iv) Prove that every group of Composite order possesses proper subgroups.

   (v) Let R and S be two rings. A homomorphism $f: R \to S$ is injective if and only if $Ker\ f = \{0\}$.

   (vi) Let R be a ring such that $x^2 = xf$ or all $x \in R$. Prove that R is a commutative ring. $x + y = 0 \Rightarrow x = y$.

   (vii) Define Nilpotent.

   (viii) State second isomorphism theorem on rings.