(c) Define random variable and probability density function.

(d) A random variable X has the following probability distribution :

Value of X :	0	1	2	3	4	5	6	7	8
p(x) :	k	3k	5k	7k	9k	11k	13k	15k	17k

Determine the value of k.

(e) The mean of the binomial distribution is 20 and standard deviation is 4. Calculate the parameter of its distribution.

(f) Differentiate between the discrete and continuous random variables.

(g) State weak law of large numbers.

(h) If X and Y be two independent random variables then find the moment generating function of X + Y .

(i) State Binomial and Poisson distribution.

(j) If the moment generating function (m.g.f.) of X is given by

$M_x(t) = \frac{1}{1-t^2}$ where $|t| < 1$. Find the m.g.f. of $Y = \frac{X-4}{4}$.

10×3=30

Roll No. ....

Total No. of Pages : 4

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E-25/2111
MATHEMATICAL FOUNDATION OF STATISTICS—1106T
Semester—I

Time Allowed : Three Hours] [Maximum Marks : 70

Note :– The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

SECTION—A

1. (a) A continuous r.v. X follows the probability law $f(x) = Ax^2$, $0 \le x \le 1$. Determine A and find the probability that X lies between 0.2 and 0.5.

(b) Let X be the random variable such that :

$P(X = -2) = P(X = -1)$, $P(X = 2) = P(X = 1)$ and

$P(X > 0) = P(X < 0) = P(X = 0)$.

Obtain the probability mass function of X and its distribution functions.

2. Two fair dice are thrown independently. Three events A, B and C is defined as follows :

A : Even face with first dice.

B : Even face with second dice.

C : Sum of the points on the two dice is odd.

Discuss the independence of events A, B and C.

3. (a) The time required to repair a machine is exponentially distributed with parameter ½. What is the probability that a repair time exceeds 2 hours ? What is the conditional probability that a repair time takes at least 10 hours given that its duration exceeds 9 hours ?

(b) In an intelligence test administered on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distributed, find the number of children who have scored (i) over 90 marks, (ii) below 40 marks and (iii) between 50 and 80 marks. Given that $P(0 < Z < 1.5) = 0.4332$; $P(Z < 1) = 0.8413$; $P(Z < 0.5) = 0.6915$.

4. A random variable X has a density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; \quad x > 0 \\ 0 & ; \quad \text{elsewhere} \end{cases}.$$

Find the moment generating function and the mean and variance.

2×10=20

SECTION—B

5. The joint probability distribution of a pair of random variable is given by the following table :

Y↓	X→	1	2	3
1		0.1	0.1	0.2
2		0.2	0.3	0.1

Find (i) the marginal distribution, (ii) conditional distribution of X given Y = 1. (iii) $P(X + Y < 4)$.

6. (a) State and Prove Bayes theorem.

(b) A man is equally likely to choose any of the three routes A, B, C from his house to the railway station, and his choice of route is not influenced by the weather. If the weather is dry, the probabilities of missing the train by routes A, B and C are respectively 1/20, 1/10 and 1/5. He sets out on a dry day and misses the train. What is the probability that the route chosen was C ?

7. Examine whether the weak law of large number holds for the sequence $\{X_k\}$ of independent random variables defined as :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$, $P(X_k = 0) = 1 - 2^{-2k}$.

8. Let X and Y be two random variables each taking three values –1, 0, 1 and having the joint probability distribution :

X Y	–1	0	1
–1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

Prove that X and Y have different expectation. Also prove that X and Y are uncorrelated.

2×10=20

SECTION—C

9. Write in brief :

(a) State the axiomatic definition of probability.

(b) Define the condition probability for an event.