**Roll No. ..........................** **Total No. of Pages : 3**

# PC 13471-NJ

**E-25/2111**

**CALCULUS—I-(1101 T)**

**Semester—I**

Time Allowed : 3 Hours] [Maximum Marks : 70

**Note :—** Candidates are required to attempt ***five*** questions in all, selecting at least ***two*** questions each from Sections A and B. Section C is compulsory.

## SECTION—A

1. (i) Using the definition of limits, show that

$$\lim_{x\to c}(x-c)\sin\frac{1}{x-c}=0.$$

(ii) Let $f(x)=\begin{cases}1, & x\le 3\\ ax+b, & 3<x<5\\ 7, & 5\le x\end{cases}$ find the constants a and b so that the function f may be continuous for all x. 5+5

2. (i) Examine for concavity upwards, concavity downwards and points of inflexion the curve $y = x^3 - 6x^2 + 9x + 1$.

(ii) Show that the asymptotes of the curve $x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$ meet the curve in three points which lies on the line $x - y + 1 = 0$. 5+5

3. (i) Sketch the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$.

(ii) Compute L(P,f) and U(P,f) for the function $f(x) = \cos x$, where $P=\left\{0,\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2}\right\}$. 5+5

4. (i) State and prove the Fundamental Theorem of Integral Calculus.

(ii) If $0 < x < 1$, then show that $\frac{x}{1-x} \geq \log(1-x)^{-1} \geq x$. 5+5

## SECTION—B

5. (i) Integrate $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$.

(ii) Integrate $\int \frac{2x}{(x^2+1)(x^2+3)}dx$. 5+5

6. (i) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

(ii) Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

5+5

7. (i) State and prove Cauchy's second theorem on limits.

(ii) Prove that the sequence $\{a_n\}$, where

$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{2n}$ is convergent. 5+5

8. (i) Discuss the convergence of the series $2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots$

(ii) Calculate the approximate value of $\sqrt{10}$ to four decimal places by taking the first four terms of an appropriate Taylor's expansion. 5+5

## SECTION—C

9. (i) Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$, for all x in [a, b], where m, n are positive integers.



(ii) Find the area of the region included between the parabola $y = \frac{3}{4}x^2$ and the line $3x - 2y + 12 = 0$.

(iii) If $f(x) = x\left[\frac{1}{x}\right]$, then show that $\lim_{x\to\frac{1}{2}} f(x)$ does not exist.

(iv) Determine the set of all values where the function $f(x) = \frac{x}{1+|x|}$ is differentiable.

(v) If [x] stands for integral part of x, then show that $\int_0^1 [5x]dx = 2$.

(vi) Show that $\lim_{n\to\infty} \frac{1}{n}[(m+1)(m+2)\ldots(m+n)]^{\frac{1}{n}} = \frac{1}{e}$, where m is fixed positive integer.

(vii) Show that the sequence $\{a_n\}$, where $a_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{2n}$ does not converge, by showing that it is not a Cauchy sequence. Prove that $\{a_n\}$ diverges to $\infty$.

(viii) Discuss the convergence or divergence of the series $\sum \frac{1}{n}\sin\frac{1}{n}$.

(ix) Show that the series $\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \cdots$ is conditionally convergent.

(x) Expand $f(x) = \cos x$ in powers of $\left(x - \frac{\pi}{2}\right)$ by Taylor's series.

10×3=30