TCS

Roll No. .... Total Pages : 5

13109/N

L–4/2111

DIFFERENTIAL EQUATIONS–II

Paper–MM-603/AMC–310

Semester–III

(Common for Maths./AMC)

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. State and prove the existence and uniqueness theorem of solutions of First order differential equation $\frac{dw}{dz} = f(z, w)$ for complex systems.

2. Let $\psi = \psi(t, r)$ be a continuous non-negative function defined on $0 < t < a$, $r \geq 0$ $(a > 0)$ and nondecreasing in r for fixed t there. Suppose that for each $\alpha, 0 < \alpha < a$, the function ρ defined by $\rho(t) = 0, 0 \le t < \alpha$ is the only differentiate function on $0 \le t < \alpha$ for which $\rho'_+(0) = \lim(\rho(0))/t$ as $t \to 0+$ exists, $\rho'(t) = \psi(t, \rho(t)), 0 < t < \alpha$ and $\rho(0) = \rho'_+(0) = 0$. Let $f \in C$ on the (n + 1)-dimensional region $R : |t - r| \le a, |x - \xi| \le b$, $a, b > 0$ and satisfy there, for $t \ne r$, $|f, (t, x) - f(t, y)| \le \psi(|t - r|, |x - y|)$, then prove that there exists at most one solution $\psi \in C^1$ of $x' = f(t, x)$ in R on $|t - r| \le a$ for which $\psi(r) = \xi$.

3. Let $f \in C(n=1)$ on the rectangular $0 \le t \le a, |x| \le b$, where a, b > 0 and assume $f(t, x_1) \le f(t, x_2)$ if $x_1 \le x_2$ and $f(t, 0) \ge 0$ for $0 \le t \le a$. Prove that the successive approximations converge to a solution of $x' = f(t, x), x(0) = 0$, on $0 \le t \le \alpha = \min\left(a, \frac{b}{M}\right)$ where $M = \max|f|$ on the rectangle.

4. State and prove Caratheodory theorem.

SECTION—B

5. Find the distribution which gives rise to the potential

$$\psi = \begin{cases} a^2 - 3x^2 & ; \; r < a \\ \dfrac{a^5\left(y^2 + z^2 - 2x^2\right)}{r^5} & ; \; r > a \end{cases}$$

where $r^2 = x^2 + y^2 + z^2$.

6. Show that the family of right circular cones $x^2 + y^2 = cz^2$, where c is a parameter, forms a set of equipotential surfaces. Also, find their corresponding potential function.

7. A point charge q is placed at a point with position vector F outside a grounded conducting sphere of radius a. Find the electrostatic potential of the field and show that the image system consists of a change $-\frac{qa}{f}$ situated at the inverse point $\frac{a^2F}{f^2}$.

8. A uniform insulated sphere of dielectric constant κ and radius 'a' carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2a^3\kappa n}{(2n+1)(\kappa n + n + 1)^2}$ to the electrostatic energy.

SECTION—C

9. Write short notes on the following :

(i) Define equipotential surface.

(ii) Write a short note on the successive approximation for the differential equation.

(iii) State only existence theorem of solutions of first order differential equation for complex systems.

(iv) State maximum and minimum solutions of differential equation $x' = f(t, x)$ with $x(\tau) = \xi$.

(v) State Kalvin's Inversion theorem.

(vi) Write a short note on Axial symmetry.

(vii) State Copson's theorem.

(viii) State Green's theorem for Laplace function.

(ix) Show that the continuity of a function f is not sufficient for the convergence of the successive approximations.

(x) How the interior Dirichlet boundary value problem for Laplace equation differ from their exterior Dirichlet boundary value problem.