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DIFFERENTIAL GEOMETRY

Paper–MATM–1104T/AMCM 1105T

Semester–I

(Common for AMC & Math.)

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. (a) Show that a local diffeomorphism $f : S_1 \to S_2$ is equireal if and only if, for any surface patch $\sigma(u, v)$ on S_1, the first fundamental forms $E_1du^2 + 2F_1dudv + G_1dv^2$ and $E_2du^2 + 2F_2dudv + G_2dv^2$ of the patches σ on S_1 and $f \circ \sigma$ on S_2 satisfy $E_1G_1 - F_1^2 = E_2G_2 - F_2^2$. 5

   (b) What is the effect of dilation of R^3 to the Gaussian and mean curvatures of a surface S? 5

2. Define a Reparametrization of a map. Prove that a parametrized curve has a unit speed reparametrization if and only if it is regular. 10

3. (a) If σ is a surface patch of an oriented surface S, then the matrix of the Weingarten map W with respect to the basis $\{\sigma_u, \sigma_v\}$ of $T_p(S)$ is $F_I^{-1}F_{II}$ where $F_I = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$ and $F_{II} = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$. 5

(b) Give the geometrical interpretation of a tangent vector. 5

4. Discuss geometrically the First Fundamental Form. If $Edu^2 + 2Fdudv + Gdv^2$ is the first fundamental form of a surface patch $\sigma(u, v)$ of a surface S, show that for a point p in the image of σ and $v, w \in T_p(S)$, we have

$$< v, w >= Edu(v)du(w) +$$

$$F[du(v)dv(w) + du(w) + dv(v)] + Gdv(v)du(w).$$

SECTION—B

5. If (τ) denotes the length of a part of a smooth family of curves between any two points on the surface patch, then the unit speed curve γ is a geodesic iff $\frac{d}{dt}(\tau) = 0$ when $\tau = 0$ for all families of curves γ^T with $\gamma^0 = \gamma$. 10

6. (a) Show that the Gaussian curvature of a surface S is preserved by local isometries. 5

(b) Prove that the Codazzi-Mainardi equations reduce to $L_v = \frac{1}{2}E_v\left(\frac{L}{E} + \frac{N}{G}\right)$ and $N_u = \frac{1}{2}G_u\left(\frac{L}{E} + \frac{N}{G}\right)$ where $Edu^2 + Gdu^2$ is the first fundamental form and $Ldu^2 + Ndv^2$ is the second fundamental form. 5

7. Every connected compact surface whose Gaussian curvature is constant is a sphere. 10

8. (a) Prove that every Helicoid is a minimal surface. 5

(b) Prove that any local isometry between two surfaces takes the geodesics of one surface to the geodesics of the other. 5

SECTION—C

9. Write short answer on the following : 10×3=30

(i) Show that the area of a surface patch is unchanged by reparametrization.

(ii) Define the collection of coordinate neighbourhoods and surface which constitute an atlas for the sphere S^2.

(iii) Define Meridians and Parallels of a surface.

(iv) Prove that a unit speed curve on a surface is geodesic iff its geodesic curvature is zero everywhere.

(v) A local diffeomorphism $f : S_1 \to S_2$ is a local isometry iff for any surface patch σ_1 of S_1, the patches σ_1 and $f \circ \sigma_1$ on S_1 and S_2 respectively have the same first fundamental form.

(vi) Differentiate between a level curve and a parametrized curve. Is parametrization of a curve unique ? Explain with the help of an example.

(vii) Find the second fundamental form of a unit cylinder $\sigma(u, v) = (\cos v, \sin v, u)$.

(viii) Find the unit speed reparametrization of the curve $\gamma(t) = (-\sin t, \cos t, 1)$ by its arc length starting from (–1, 0, 1).

(ix) State and prove Geodesic equations.

(x) Show that if the tangent vector of parameterized curve is constant, the image of the curve is a part of the straight line.