PC 12971-N

K-8/2111

MATHEMATICS FOR CHEMISTS—1104T

Semester—I

Time Allowed : 3 Hours] [Maximum Marks : 55

Note :— The candidates are required to attempt ***two*** questions each from Sections A and B. Section C will be compulsory.

SECTION—A

1. (a) Evaluate divergence of the function $F = x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}$ at the point (1, 2, 3).

   (b) Find the value of λ, so that $\vec{a}\,(\vec{b}\times\vec{c}) = 0$, where

   $$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k},\quad \vec{b} = \hat{i} - \lambda\hat{j} + \hat{k},\quad \vec{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}\,.$$

2. Verify the Cayley Hamilton theorem for the matrix

   $$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}.$$

3. Find λ and μ so that the system of linear equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) infinite number of solutions, (iii) a unique solution

4. (a) Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\dfrac{x+3}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$.

   (b) If $A + B = \dfrac{\pi}{4}$ then find the value of tan A + tan B + tan A tan B.

2×8=16

SECTION—B

5. (a) If $u = \sin^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$.

(b) Evaluate $\int \frac{x}{(x-1)(x+3)}\,dx$ by using partial fraction.

6. If the function $f(x) = \begin{cases} 3ax+b & ; \quad x>1 \\ 11 & ; \quad x=1 \\ 5ax-2b & ; \quad x<1 \end{cases}$ is continuous at $x = 1$

find the values of a and b.

7. Solve in series the equation using :

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0.$$

8. (a) Solve the differential equation :

$$(3x^2-y^2)dy-2xy\,dx=0.$$

(b) Solve the differential equation :

$$\frac{dy}{dx}=\frac{y-x}{y+x}$$

2×8.5=17

SECTION—C

9. Do briefly :

(a) Find the value of $\sin 15^{\circ}$.

(b) Find the dot product of the vectors $2\hat{i}+3\hat{j}-5\hat{k}$ and $\hat{i}-2\hat{k}$.

(c) Define dot and cross products for two vectors.

(d) Define Symmetric and Hermitian matrices.

(e) If $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$. Find 2A – B.

(f) Define eigen values and eigen vectors.

(g) State the conditions for finding the maximum and minimum of functions of two variables f(x, y).

(h) Check whether the function :

$$f(x) = \begin{cases} 1+x \; ; & x\le 2 \\ 5-x \; ; & x>2 \end{cases}$$

is differential or not at x = 2 ?

(i) Evaluate $\int x\sin x\,dx$.

(j) Check whether the equation :

$$(1+2xy\cos x^2-2xy)dx+(\sin x^2-x^2)dy=0$$

is exact or not ?

(k) Find the equation of the line which passes through the point (2,1,3) and is parallel to the vector $2\hat{i}+3\hat{j}-4\hat{k}$. 11×2=22