Roll No. .... Total Pages : 4

11762/NJ

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DIFFERENTIAL EQUATIONS

Paper–232

Semester–III

Time Allowed : 3 Hours] [Maximum Marks : 45

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 7 short answer type questions carrying 3 marks each.

SECTION—A

1. Solve in series :

$$2x\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}-2y=0. \qquad 6$$

2. Prove that :

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx=0,\ \text{if}\ m\neq n. \qquad 6$$

3. (a) Show that the following equation can be expressed as Sturm-Liouiville equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-k^2\right)y=0. \qquad 3$$

(b) Show that $XF\left(\frac{1}{2},\frac{1}{2},\frac{3}{2};x^2\right)=\sin^{-1}x.$ 3

4. Solve for complete solution by Charpit's method :

$$z=p^2x+q^2y. \qquad 6$$

SECTION—B

5. Find the general solution of

$$\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=2x+3y. \qquad 6$$

6. Solve by Monge's method of the equation :

$$2x^2r-5\times y\,S+2y^2t+2(px+qy)=0. \qquad 6$$

7. Solve $\left(D^3 - 7dDD'^2 - 6D'^3\right)Z = \cos(x-y) + e^{x+2y}$. 6

8. (a) Find the Laplace transform of

$$g(t) = \begin{cases} 0 & , \quad 0 < t < 2\pi \\ \sin t & , \quad t > 2\pi \end{cases}$$

by using Second shifting theorem. 3

(b) Evaluate $L\left(e^{-7t}t^{1/2}\right)$. 3

SECTION—C

9. Attempt all the following parts : 7×3=21

(i) Find the Laplace transform of $4e^{-5t} - 3e^{2t}$.

(ii) Find the general solution of the following partial differential equation :

$$\left(D^3 - 6D^2D' + 11DD'^2 - 6D'^3\right)Z = 0.$$

(iii) Solve the partial differential equation :

$$P + q = Pq.$$

(iv) Using Convolution theorem find the inverse Laplace of the function $\frac{1}{(s+\alpha)(s+\beta)}$.

(v) Find the Regular and Singular points of the differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

(vi) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

(vii) Verify that Legendre polynomial :

$P_4(x) = \frac{35}{8}x^4 - \frac{30}{8}x^2 + \frac{3}{8}$ satisfies Legendre's equation, when n = 4.