

Roll No.

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11752/NJ**D-1/2111****TRIGONOMETRY AND
DIFFERENTIAL CALCULUS**

Paper-1102T

Semester-I

Time Allowed : 3 Hours] [Maximum Marks : 45

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 7 short answer type questions carrying 3 marks each.

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SECTION—A

1. Solve the equation $x^{12} - 1 = 0$ and find the roots satisfy the equation $x^4 + x^2 + 1 = 0$. 6
2. If $\tan(\theta + i\phi) = \sin(x + iy)$, prove that : $\coth y \cdot \sinh 2\phi = \cot x \cdot \sin 2\theta$. 6
3. State and prove De-Moivre's theorem. 6
4. Sum upto infinity : 6

$$\frac{1}{2} \sin a + \frac{1.3}{2.4} \sin 2a + \frac{1.3.5}{2.4.6} \sin 3a + \dots$$

SECTION—B

5. If $y = x^2 e^x$, then prove : 6

$$y_n = \frac{1}{2} n(n-1) y_2 - n(n-2) y_1 + \frac{1}{2} (n-1)(n-2) y.$$
6. Find the values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward or downward. Also determine points of inflexion. 6

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7. Show that asymptotes of curve : 6

$$x^3 + 2x^2y - xy^2 + 2y^3 + 4y^2 + 2xy + y - 1 = 0.$$

meet the curve in three points which lies on the line $x + 3y = 1$.

8. Find the centre of curvature at any point (x, y) of the parabola $y^2 = 4ax$. Also find its evolute. 6

SECTION—C

9. Explain the following : 7×2=14

(i) Prove that one of the values of

$$(a + ib)^{\frac{m}{n}} + (a - ib)^{\frac{m}{n}} \text{ is :}$$

$$2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n} \tan^{-1} \frac{b}{a}\right).$$

(ii) Show that $i \log \left(\frac{x-i}{x+i} \right) = \pi - 2 \tan^{-1} x$.

(iii) Prove $\tan^{-1} \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) = n\pi + \frac{\pi}{4} + \tan \theta$

$$-\frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta.$$

(iv) Show that $\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$.

(v) Find nth derivative of $e^x \log x$.

(vi) Prove that curvature of straight line is zero.

(vii) Find position and nature of :

$$x^4 + y^3 + 2x^2 + 3y^2 = 0.$$