Roll No. .... Total Pages : 4

11751/NJ

O–1/2111

ALGEBRA

Paper–1101T

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 45

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 7 short answer type questions carrying 3 marks each.

SECTION—A

1. Using elementary operations only, find the inverse of the matrix : 6

$$\begin{bmatrix} 1 & -2 & 3 \\ -2 & -1 & 0 \\ 4 & -2 & 5 \end{bmatrix}.$$

2. Prove that every square matric A over C can be expressed uniquely as P + iQ, where P and Q are Hermitian matrices. 6

3. Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, given that one root is negative of the other. 6

4. Solve the Cubic $x^3 - 15x - 126 = 0$ by Cardan's method. 6

SECTION—B

5. Check whether the matrices $\begin{bmatrix} 1 & -3 & 1 \\ 6 & -7 & 8 \\ 2 & 1 & -4 \end{bmatrix}$ and

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are equivalent or not ? 6

6. Find the eigenvalues and the corresponding eigen vectors for the matrix : 6

$$\begin{bmatrix} 3 & 2 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

7. Determine λ for which the following system of equations is consistent and find the solutions for all such value of λ: 6

$3x - 4y + 2z = 0;\ x + y - 2z = 0;\ 5x - 2y - 2z = \lambda.$

8. Verify Cayley Hamilton theorem for the matrix:

$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$ 6

SECTION—C

9. Write short notes on the following: 7×3=21

(i) Show that $B^\theta AB$ is Hermitian or skew Hermitian according as A is Hermitian or skew Hermitian.

(ii) State Descarte's rule of signs.

(iii) If A is n-rowed square matrix of rank n – 1, then show that $\operatorname{adj} A \neq 0$.

(iv) Define eigenvalue and eigenvector.

(v) Prove that square matrix and its transpose matrix has the same set of eigenvalues.

(vi) For a system of linear equation AX = B, state the conditions for which this system of equation has (i) no solution and (ii) infinite many solutions.

(vii) Construct a cubic polynomial f(x) having the properties f(x) is monic, f(0) = – 8, f(4) = 0 and sum of roots of f(x) = 0 is 7.