

Roll No. ....

Total No. of Pages : 3

**PC 11477-NH**

CS/2111

**DISCRETE MATHEMATICS—I, Paper—II**

**Semester—V**

Time Allowed : Three Hours]

[Maximum Marks : 40

**Note :-** The candidates are required to attempt *two* questions each from Sections A and B. Section C will be compulsory.

**SECTION—A**

- I. (a) Prove that product of two lattices is a lattice. 3
- (b) Find the number of different signals that can be made by arranging at least three flags in order on a vertical pole, if 6 different flags are available. 3
- II. (a) In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find
- (i) How many drink both tea and coffee.
- (ii) How many drink coffee but not tea. 3
- (b) Use Pigeonhole Principle to find how many people among 200000 people are born at same time (hour, minute, second) ? 3

III. (a) Define Partial Order Relation and prove that if  $X = \{1, 2, 3\}$  and  $P(X) =$  Power Set of  $X$  then prove that  $\subseteq$  is a partial order relation in  $P(X)$ . 3

(b) Prove that intersection of two equivalence relations on a non-empty set is again an Equivalence relation. 3

IV. (a) Find the number of arrangements of the word INDEPENDENCE. In how many of these arrangements do the words start with P ? 3

(b) Discuss the types of Grammars. 3

#### SECTION—B

V. (a) Discuss Travelling Salesman Problem using graph theory. 3

(b) State and prove Euler's formula for planar graph. 3

VI. (a) A three-state finite state machine has  $\{0, 1\}$  as its input and output alphabets. Given the following input sequence and its corresponding output sequence, determine the machine.

Input sequence : 00010101

Output sequence : 011001110. 3

(b) Explain how finite state machines can be used as a device to recognize sentence in a language. 3

VII. (a) Show that finite connected graph is Eulerian if and only if each vertex has even degree. 3

(b) Show that the graph  $K_5$  is not a planar graph. 3

VIII. (a) Prove that the number of edges in a complete graph with  $n$  vertices is  $\frac{n(n-1)}{2}$ . 3

(b) Prove that a graph is connected if and only if it has a spanning tree. 3

#### SECTION—C

IX. (a) Write a short note on Principle of Inclusion and Exclusion.

(b) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) No girl

(ii) At least one boy and one girl ?

(c) Define an equivalence relation.

(d) State Pigeon-Hole Principle.

(e) Show that there is one and only one path between every pair of vertices in a tree.

(f) Show that  $K_{3,3}$  satisfies the inequality  $|E| \leq 3|V| - 6$ .

(g) State and prove Handshaking theorem.

(h) Explain Dijkstra's Algorithm to find the shortest path between two vertices in a weighted graph.  $8 \times 2 = 16$