PC-11791/NJ

K-7/2111

ANALYSIS-I -301

Semester- III

(Common for MC and B.Sc. Hons. in Mathematics)

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *five* questions in all, select *two* questions each from Section A and B carrying 20 marks each and Section C is compulsory consisting of 10 short answer type questions carrying 3 marks each.

SECTION – A

I. Show that the smallest member of a set, if it exists, is the infimum of the set. 10

II. If f : A $\rightarrow$ B is one-to-one and B is countable then A is countable. 10

III. State and prove the Lindel of covering theorem. 10

IV. State and prove the Bolzano Weierstrass theorem. 10

SECTION – B

V. Show that the image of a Cauchy sequence under a uniformly continuous function is again a Cauchy sequence. 10

VI. Let (X, d) be any metric space. Show that the function d_1 defined by

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \quad \forall\ x, y \in X$$

in a metric on X. 10

VII. Show that the limit of a function is unique. 10

VIII. Let A be a connected subset of a metric space X, and let B be a subset of X such that $A \subseteq B \subseteq \overline{A}$, then show that B is also connected. 10

SECTION – C

IX. (i) Define the supremum and infimum of a set.

(ii) Define the Archimedean property of real numbers.

(iii) Define countable and uncountable sets with examples.

(iv) State the Cantor intersection theorem.

(v) State the Heine-Borel covering theorem.

(vi) Show that every open sphere is an open set.

(vii) Define a connected set with an example.

(viii) Define the components of a metric space.

(ix) Explain the uniform continuity of a function.

(x) Show that a non-empty connected subset of a metric space X is a component, if it is both open and closed.

(10×3=30)