BS-2051

728|MH

Subject: Analysis-II **Paper-V**

Title of the Paper: Analysis-II (SEM-IV)

Time Allowed: 3 Hrs. **Maximum Marks: 40**

Note: The candidates are required to attempt two questions each from the Section A & B.

Section C is compulsory.

Section-A

I. Define the Point-wise and Uniform convergence of sequence of functions. Show that the sequence $f_n(x) = x^n$ is uniformly convergent on [0, k], $k < 1$ and only pointwise convergent on [0, 1]. (6)

II. Show that the series $\sum x\, e^{-nx}$ does not converge uniformly in [0, 1]. (6)

III (a) Show that the series, for which $f_n(x) = \frac{nx}{1+n^2x^2}$, $0 \leq x \leq 1$ cannot be differentiated term by term at x = 0. (3)

(b) Test the uniform convergence of $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$ in [− 1, 1]. (3)

IV. (a) Prove that the power series $1 + \frac{a.b}{1.c}x + \frac{a(a+1).b(b+1)}{1.2\ c(c+1)}x^2 + \ldots$ has unit radius of convergence. (3)

(b) Show that the power series $\sum a_n x^n$ either

(i) converges for *all* values of x

(ii) converges *only* for $x = 0$

(iii) converges for x in *some* region on the real line

by giving example in each of the above three cases. (3)

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Section-B

V. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,\hat{\imath} + (2xz - y)\,\hat{\jmath} + z\,\hat{k}$ along

(i) the straight line from (0, 0, 0) to (2, 1, 3).

(ii) the curve defined by $x^2 = 4y,\ 3x^3 = 8z$ from $x = 0$ to $x = 2$. (6)

VI. Show that $\iint \vec{F}.\hat{n}\, dS = \frac{3}{2}$; where $\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (6)

VII. State the Gauss Divergence theorem of vector calculus. Use it to evaluate $\int \vec{A}.\vec{dS}$, where $\vec{A} = x^3\,\hat{\imath} + y^3\hat{\jmath} + z^3\,\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. (6)

VIII. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\,\hat{\imath} + (yz + 4)\,\hat{\jmath} - (xz)\hat{k}$ over the surface of a cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above XOY plane. (6)

Section-C

IX.

(i) Show that 0 is a point of non-uniform convergence of the sequence $f_n(x) = nxe^{-nx^2}$, where $x \in R$.

(ii) Define uniform convergence of series of functions.

(iii) Differentiate between radius of convergence and interval of convergence of a power series.

(iv) Find the radius of convergence and interval of convergence of the series $\sum_{n=3}^{\infty} \frac{x^{n-3}}{(n-3)!}$.

(v) Evaluate Curl Curl of $\vec{V} = (2xz^2)\hat{\imath} - (yz)\,\hat{\jmath} + (3xz^3)\,\hat{k}$ at (1, 1, 1).

(vi) The accelerating of a particle at any time t is given by $\vec{a} = 18\cos 3t\,\hat{\imath} - 8\sin 2t\,\hat{\jmath} + 6t\,\hat{k}$. If the velocity $\vec{v}$ and displacement $\vec{r}$ be zero at t = 0, then find $\vec{v}$ and $\vec{r}$ at any time t.

(vii) State Green's theorem of vector calculus.

(viii) If $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, then find grad $\vec{r}$ and grad $\left(\frac{1}{\vec{r}}\right)$. $2 \times 8 = 16$

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