Roll No. .... Total Pages : 4

4421/MH

C-2051

ALGEBRA-II

Paper–III

Semester–VI

Time allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt two questions each from section A and section B carrying 6 marks each and the entire Section C consisting of 8 questions carrying 2 marks each.

SECTION-A

1. (a) Show that the set $B = [1, x, x^3 ....., x^m]$ of m+1 polynomials is a basis set for the vector space of polynomial of degree m over R. 3

   (b) Show that the vectors (1, 2, 3), (0, 1, 2) and (0, 0, 1) generates $V_3(R)$. 3

2. Let V(F) is finitely generated vector space, prove that any maximal linearly independent subset of V is a basis of V. 6

3. If v_1, v_2 are finite dimensional subspaces of a finite dimensional vector space V(F). Prove that $v_1 + v_2$ is also finite dimensional and

   $\dim(v_1 + v_2) = \dim v_1 + \dim v_2 - \dim(v_1 \quad v_2)$. 6

4. Prove that any linearly independent set in V(F) can be extended to a basis of V. 6

SECTION-B

5. Let V = R^3 and at Tiv V be the linear transformation defined T(x, y, z) = (2x, 4y, 5z). Find the matrix of T w.r.t. basis $(\frac{2}{3}, 0, 0), (0, \frac{1}{2}, 0)$ and $(0, 0, \frac{1}{4})$. 6

6. Prove that if V(F) and W(F) are finite dimensional, then the vector space of all linear transformations from V to W is also a finite dimensional vector space and its dimension is equal to (dimV)(dimW). 6

7. Let T be a linear operator on IR^3 defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. 6

8. Prove that characteristics and minimal polynomial of an operator of matrix have same irreducible factors. 6

SECTION-C

9. (i) Write $(1, -2, 5)$ as a linear combination of the vector $(1, 1, 1), (1, 2, 3), (2, -1, 1)$.

(ii) Prove that superset of linearly dependent set of vectors is linearly dependent.

(iii) Define co-ordinate vector relative to the basis S of vector space V(F).

(iv) Let W be subspace of vector space $V_3(R)$ generated by $\{(1, 0, 0), (1, 1, 0)\}$. Find $\frac{V}{W}$ and its basis.

(v) Let $T : V \quad W$ be a linear transformation. Prove that range T is a subspace of W(F).

(vi) Define a singular transformation.

(vii) Prove that inverse of invertible operator is unique.

(viii) Find characteristic polynomial of $T : R^2$ R^2 defined by $T(x, y) = (x + 2y, 3x + 2y)$.

$2 \times 8 = 16$