PC-4362/MH

B/2051

ANALYSIS–II

Paper–V

(Semester–IV)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all selecting *two* questions each from Section A and Section B and compulsory question of Section C.

SECTION–A

I. State and prove Cauchy's criterion for uniform convergence of a sequence of functions. 6

II. (a) Does pointwise convergence imply uniform convergence? Justify your answer. 3

(b) Show that

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\ldots\ldots, \ -1 \le x < 1.$$

3

III. (a) Show that the power series

$$1 + \frac{\alpha.\beta}{1.\gamma}x + \frac{\alpha.(\alpha+1).\beta.(\beta+1)}{1.2\gamma.(\gamma+1)}x^2 + \ldots\ldots\ldots \infty \text{ has}$$

unit radius of convergence. 3

(b) Show that the sequence $\{f_n\}$, where $f_n(x) = \dfrac{1}{x+n}$ is uniformly convergent in any interval $[0, b]$, $b > 0$. 3

IV. (a) Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{\alpha_n x^{2n}}{1+x^{2n}}$ is uniformly convergent for all real x if $\displaystyle\sum_{n=1}^{\infty} a_n$ is absolutely convergent. 3

(b) Show that the series, for which

$$f_n(x) = \frac{nx}{1+x^2 n^2}, \; 0 \le x \le 1$$

cannot be differentiated term by term at $x = 0$. 3

SECTION–B

V. Verify Green's theorem in the plane for $\displaystyle\oint_C (3x^2 - 8y^2)\, dx + (4y - 6xy) dy$, where C is the boundary of the region bounded by the parabolas $y = x^2$ and $x = y^2$. 6

VI. Show that the integral $\displaystyle\oint_C (2xy + 3)dx + (x^2 - 4z)dy - 4ydz$, where C is any path joining (0, 0, 0) to (1, –1, 3) does not depend on the path C and evaluate the integral. 6

VII. Show that $\iint_S \vec{F}.\hat{n}\, ds = \frac{3}{2}$; where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$, $z = 1$. 6

VIII. (a) Show that Curl $\left(\phi\, \vec{A}\right) = \phi\left(\text{curl } \vec{A}\right) + \left(\text{grad } \phi\right) x\ \vec{A}$. 3

(b) Show that

$$\nabla^2 r^n = n(n+1)r^{n-2}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 3

SECTION–C

IX. (a) Find the radius of convergence of the series :

$$\sum \frac{(n+1)}{(n+2)(n+3)} x^n$$

(b) State Gauss Divergence Theorem.

(c) Show that the series $\sum_{n=0}^{\infty} (-1)^n x^n$, $0 \le x \le 1$ admits of term by term integration on [0, 1], though it is not uniformly convergent on [0, 1].

(d) State Weierstrass M-Test for uniform convergence of sequence of functions.

(e) Verify the formula $\frac{d}{dt}\left(\vec{a}\,.\,\vec{b}\right) = \vec{a}.\frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt}.\vec{b}$

for the vectors $\vec{a} = 5t^2\hat{i} + t\,\hat{j} - t^3\hat{k}$,

$\vec{b} = \sin t\;\hat{i} - \cos t\;\hat{j}$.

(f) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that the vector $r^{-3}\,\vec{r}$ is solenoidal.

(g) Find the directional derivative of the function $xy^2 + yz^3$ at the point (2, –1, 1) in the direction of outward normal to the surface $x \log z - y^2 + 4 = 0$ at (–1, 2, 1).

(h) Show that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots, -1 \le x \le 1.$$

(2×8=16)