Roll No. .... **Total Pages : 3**

4251/MJ

G-11/2051

NUMBER THEORY

Paper–BHM-601

Semester–VI

Time allowed : 3 Hours] [Maximum Marks : 70

Note : The candidates are required to attempt two questions each from section A and section B carrying 10 marks each and the entire Section C consisting of 10 questions carrying 3 marks each is compulsory.

SECTION-A

1. If P_n is the nth prime number then Prove that $P_n \quad 2^{2^{n-1}}$.

2. State and Prove Fundamental theorem of arithmetic.

3. (a) If $ca = cb$ (mod n), then $a = b$ (mod n/d), where $d = \gcd(c, n)$.

   (b) By using the definition of Congruence show that 41 divides $2^{20} - 1$.

4. (a) State and prove Mobius Inversion Formula.

   (b) State and Prove Euler's theorem.

SECTION-B

5. State and prove Euler's Criterion of Quadratic Residues.

6. Find the number of Farey fractions a/b of order n satisfying the inequalities $0 \quad a/b \quad 1$.

7. Define Pell's equation. Prove that if d is a positive integer not a perfect square, then $h_n^2 - dk_n^2 = (-1)^{n-1} q_{n-1}$ for all the integers $n \quad -1$.

8. Prove that the continued fraction expansion of the real quadratic irrational number 'a' is purly

periodic iff $a > 1$ and $-1 < a^* < 0$. where a^* is the conjugate of a.

SECTION-C

9. (i) Find the G.C.D of 117 and 45.

(ii) Find the index of 5 relative to each of the primitive roots of 13.

(iii) Show that 125671221 is divisible by 9.

(iv) Find the remainder when 2(28!) is divided by 31.

(v) State Chinese remainder theorem.

(vi) Find the solution of $x^2 \quad 5(mod 29)$.

(vii) Find all the quadratic residue of 13.

(viii) Evaluate n of Gauss Lemma for (5/19).

(ix) Find the value of Jacobi Symbol $\left(\frac{22}{105}\right)$.

(x) Define rational approximations & give example.