Roll No. .... Total Pages : 5

7273/N

J–15/2110

TOPOLOGY–I

Paper–MM 403/AMC 103

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : Attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. (a) Define well ordered set. Prove that the set of non-negative integers is well ordered. 5

   (b) Prove that there is no cardinal number between $\aleph_0$ and c. 5

2. (a) Prove that B is a basis for a topology τ on X if and only if for each $G \in \tau$ and each $x \in G$ there is a $U \in B$ such that $x \in U \subset G$. 5

   (b) Prove that a set D is dense in X if and only if each non-empty basic open set in X contains an element of D. 5

3. (a) Let X be a set and $\beta : P(X) \to P(X)$ be a map such that

   1. $\beta(\phi) = \phi$
   2. $\beta(A) = \beta(C(A)$
   3. $\beta o \beta(A) \subset \beta(A)$
   4. $A \cap B \cap \beta(A \cap B) = A \cap B \cap (\beta(A) \cup \beta(B))$.

   Then $\tau = \{C(A \cup \beta(A)) : A \in P(X)\}$ is a topology and Fr(A) = β(A). 5

   (b) Let Y be a subspace of X. If A $\subset$ Y is closed in Y, and Y is closed in X, then A is closed in X. 5

4. (a) Let $f : X \to Y$ be open map. Show that if A = f^{-1}(B) for some $B \subset Y$, then $f|A$ is also an open map into B. 5

(b) Let $f : X \to Y$ be a closed map. Given any subset S of Y and any open set U containing $f^{-1}(S)$, there exists an open set V containing S such that $f^{-1}(V) \subset U$. 5

SECTION—B

5. (a) Let $\{Y_\alpha : \alpha \in A\}$ be any family of spaces and $f : X \to \prod_\alpha Y_\alpha$ be a map. Then f is continuous if and only if $p_\beta o f$ is continuous for each $\beta \in A$. 5

(b) The Cartesian product topology in $\prod_\alpha Y_\alpha$ is the smallest topology for which all projections $p_\beta : \prod_\alpha Y_\alpha \to Y_\beta$ are continuous. 5

6. (a) X is locally connected if and only if the components of each open set are open sets. 5

(b) Prove that the union of any family of connected subsets having at least one point in common is also connected. 5

7. (a) If X is a non-empty compact Hansdorff space having no isolated points, then X is uncountable. 5

(b) Any locally compact space X can be imbedded in a compact space $\hat{X}$ so that $\hat{X} - X$ is a single point. Any two compact spaces having this property are homeomorphic to each other. 5

8. (a) Show that every compact metrizable space has a countable basis. 5

(b) Let $P : X \to Y$ be a closed continuous surjective map such that $p^{-1}(y)$ is compact for each $y \in Y$. Show that if X is second countable, then so is Y. 5

SECTION—C

9. Attempt all questions :

(a) How many distinct topologies can a set of three elements have? What is their partial ordering?

(b) Describe the open sets if all straight lines in the plane parallel to the x-axis are used for sub-basis.

(c) Prove that G is open in X if and only if $C(G \cap C(A)) = C(G \cap A)$ for every $A \subset X$.

(d) Give an example to show that a continuous open map need not map the interior of a set onto the interior of the image.

(e) Prove that $\aleph_0 . \aleph_0 = \aleph_0$.

(f) Let A and B be disjoint compact subspaces of the Hausdorff space X. Show that there exist disjoint open sets U and V containing A and B respectively.

(g) The continuous image of a connected space is connected.

(h) Prove that $\prod A_\alpha$ is dense in $\prod y_\alpha$ if and only if $A_\alpha \subset Y_\alpha$ is dense.

(i) Prove that the subspace of a Lindelof space need not be Lindelof.

(j) Prove that subspace of a Hausdorff space is Hausdorff'. 10×3=30