

Roll No.

Total Pages : 3

7272/N

J-15/2110

MATHEMATICAL ANALYSIS

Paper-MM-402/AMC-102

Semester-I

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : Attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. State and prove Implicit function theorem. 10
2. If a vector space X is spanned by n vectors, then $\dim X \leq n$. 10

7272N/181/W/1,310

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3. Show that the outer measure of an Interval is its length. 10
4. The family \mathbf{M} of measurable sets is an algebra of sets. 10

SECTION—B

5. State and prove Monotone convergence Lemma. 10
6. State and prove Vitali's lemma. 10
7. If f is of bounded variation on $[a, b]$, then $f'(x)$ exists almost everywhere on $[a, b]$. 10
8. State and prove Lebesgue convergence theorem. 10

SECTION—C

9. (i) If A is countable, then $m^*A=0$.
(ii) Give an example of a non-measurable set.

7272N/181/W/1,310

2

- (iii) State and prove contraction principle.
- (iv) Show that a function of bounded variation can be written as a difference of two monotone functions.
- (v) If f is bounded measurable function defined on E , then $\left| \int_a^b f \right| \leq \int_a^b |f|$.
- (vi) Show that Union of two measurable sets is measurable.
- (vii) Let E be a measurable set and for $\epsilon > 0$, then there exists an open set O , s.t. $E \subset O$ with $m^*(O \setminus E) < \epsilon$.
- (viii) If f is measurable function and $f = g$ almost everywhere, then g is also measurable.
- (ix) Show that sum of two simple functions is also simple.
- (x) State and prove Jensen's inequality.

10×3=30