

**J-13/2110****MATHS FOR CHEMISTS****Paper-104A**

(For student without Mathematics in B.Sc.)

Time : Three Hours]

[Maximum Marks : 55

**Note** : Attempt *two* questions each from Section A and B. Section C will be compulsory.

**SECTION-A**

I. Evaluate divergence and curl of the function

$$F = x^2 yz\hat{i} + xy^2 z\hat{j} + xyz^2\hat{k} \text{ at the point } (1, 2, 3).$$

II. Find all the eigen values of the matrix A over real R, where

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}, \text{ and hence verify the Cayley Hamilton}$$

theorem for the matrix A.

III. Show that the only real value of A for which the following system of equations has a non-zero solution is 6 :

$$x + 2y + 3z = \lambda x ; 3x + y + 2z = \lambda y ; 2x + 3y + z = \lambda z.$$

- IV. Find the Cartesian equation of the line which passes through the point  $(-2, 4, -5)$  and parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}.$$

### SECTION-B

- V. If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ , prove that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ .
- VI. (a) Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{x^2 - x - 6}{x^2 - 2x - 3}; & x \neq 3 \\ \frac{5}{3}; & x = 3 \end{cases}$$

at the point  $x = 3$ .

- (b) Find the maximum and minimum values, if any, of the function  $f(x) = (2x - 1)^2 + 3$ .

- VII. Solve in series the equation using :

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0.$$

VIII. Find a Fourier series to represent  $f(x) = x^2$  from  $x = -\pi$  to  $x = \pi$ .

### SECTION-C

IX. Do briefly.

(a) Find the dot product of the vectors  $2\hat{i} - 5\hat{k}$  and  $4\hat{i} + 7\hat{j} - 3\hat{k}$ .

(b) Find the value of  $\vec{a} \cdot (\vec{b} \times \vec{c})$ , where

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \quad \vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}, \quad \vec{c} = 3\hat{i} - \hat{j} - \hat{k}.$$

(c) Define Symmetric and skew-symmetric matrices.

(d) If  $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$ . Find  $2A + 3B$ .

(e) State Cayley-Hamilton theorem.

(f) State the conditions for finding the maximum and minimum of functions of two variables.

(g) Check whether the function

$$f(x) = \begin{cases} 1 + x; & x \leq 2 \\ 5 - x; & x > 2 \end{cases} \text{ is differential or not at } x = 2?$$

(h) Evaluate:  $\int x^2 e^x dx$ .

(i) Check whether the equation

$$x(x^2 + y^2 - a^2)dx + y(x^2 - y^2 - b^2) dy = 0$$

is exact or not?

(j) Define the linear differential equation with example,

(k) Write the trigonometric functions of sum and differences of angles.

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