8403/NH

A-2110
LINEAR ALGEBRA-III
SEMESTER-I

TIME 3 HOURS MM: 40

Note:- The candidates are required to attempt **two** question each from Section **A** and **B** carrying marks 6 each and entire Section C consisting of **8** questions carrying **2** marks each.

SECTION A

1. A) Investigate the values of a, b the following equations

$$x - 2y + 3z = 1, x + y - z = 4, 2x - 2y + az = b$$

have 1) No solution 2) Unique solution 3)Infinite number of solutions.

b) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. Is it diagonalisable? Justify

2. A) Determine the following matrices have same column space or not

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{bmatrix}.$$

b) Prove that characteristic roots of a unitary matrix are of unit modulus.

3. A) Find modal matrix of the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$.

B) Find PAQ form if A = $\begin{bmatrix} 2 & 3 & 4 \\ 6 & 3 & 1 \end{bmatrix}$.

4. A) Solve the system of equation $x + 2y - 2z + 2s - t = 0, x + 2y - z + 3s - 2t = 0,\ 2x + y - 7z + s + t = 0$.

b) Examine weather $(1, -3, 5)$ belongs to the linear space generated by S, where $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$ or not?

SECTION B

5. a) State and prove Extension theorem.

b) Examin whether $(1, -3, 5)$ belongs to the linear space generated by S, where $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$ or not ?

6. A) Let M and N be sub-space of R^4 defined as $M = \{(a, b, c, d): a + c + d = 0\}$, $N = \{(a, b, c, d): a = b, d = 2c\}$ Find the dimension and basis of M, N and $M \cap N$.

b) Extend $\{(-1,2,5)\}$ to two different basis of R^3.

7. A) Find linear transformation $T: R^2 \to R^2$ such that $T(2,3) = (1,2)$ and $T(,2) = (2,3)$.

b) Find range, rank , null space and nullity for $T(x, y) = (x + y, x - y, y)$ transformation on a vector space R^2.

8. A) Let $T: R^4 \to R^3$ be a linear transformation defined by $T(x_1, x_2, x_3, x_4) = (x_1 - x_2 + x_3 + x_4, x_1 + 2x_3 - x_4, x_1 + x_2 + 3x_3 - 3x_4)$ for $x_1, x_2, x_3, x_4 \in R$. Find the basis and dimension of i) Range of T ii) Null space of T. Also verify $Rank(T) + nullity(T) = dim\ (R^4)$.

b) Let V be vector space of 2 X 2 matrices of R and W be asset of all 2 X 2 diagonal matrices over R. Show that W is a subspace of V and find basis of V/W.

Section C

9)

a. Define rank of matrix
b. Show that if A is skew hermitian then so is $A - A^t$.
c. Show that if A is hermitian then what can you say about iA^t.
d. Using Cayley Hamilton theorem find A^8 if = $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$.
e. Define Diagonalizable matrix.
f. Show that all polynomials over R with no constant term forms a vector space.
g. Show that set containing vector 0 is always linear dependent
h. Define Vector space and Subspace.