Roll No. .... Total Pages : 4

7271/N

J–15/2110

ALGEBRA–I

Paper–MM 401/AMC 101

Semester–I

Time Allowed : 3 Hours] [Maximum Marks : 70

Note : Attempt **two** questions each from Sections A and B carrying 10 marks each and the entire Section C consisting of 10 short answer type questions carrying 3 marks each.

SECTION—A

1. (a) Show that a group G is solvable if and only if G has a normal series with abelian factors.

   (b) Show that every homomorphic image of nilpotent group is nilpotent.

2. Prove that alternating group A_n is simple for $n > 4$. Also prove S_n is not solvable for $n > 4$.

3. Show that there are 420 elements in S_7 having disjoint cyclic decomposition of the type (a, b) (c, d, e). Find also the number of elements in the orbit of (a, b) (c, d, e).

4. (a) Let G be a group containing an element of finite order and exactly two conjugate classes. Then show that order of G is 2.

   (b) Prove that a finite group G of order p^n (p-prime) has a nontrivial centre Z(G).

SECTION—B

5. State and prove fundamental theorem of finitely generated abelian groups.

6. Show that all the endomorphisms of an abelian group forms a ring with unity.

7. (a) Show that any ideal of matrix ring R_n is of the form A_n where A is an ideal of a ring R with unity.

(b) Prove that ideal $(x^3 + x + 1)$ is the polynomial ring $\frac{z}{(2)}[x]$ over $\frac{z}{(2)}$ is a prime ideal.

8. (a) Show that all Sylow p-subgroups of finite group G are conjugate.

(b) Show that a finite abelian group that is not cyclic contains a subgroup of type (p, p).

SECTION—C

9. Attempt all questions :

1. Write down all the homomorphism images of Klein four-group.
2. Distinguish between normal and subnormal series with suitable examples.
3. Show that every nilpotent group is solvable.
4. Show that symmetries of a rectangle form a Klein 4-group.
5. Explain stabilizer and orbit of an element of a group.

6. Find all non-isomorphic abelian groups of order 180.
7. Prove that there is no simple group of order 56.
8. Find all the left and right ideals of a ring $\begin{bmatrix} Q & Q \\ 0 & 0 \end{bmatrix}$.
9. Find the field of quotients of ring of gaussian integers Z[i].
10. Determine idempotent and nilpotent elements of ring Z/(4).