C/2110
MATHEMATICAL METHODS-I, OPT-I
SEMESTER-V (SYLLABUS DECEMBER, 2019)

TIME ALLOWED 3 Hrs

M.M 40

NOTE: **The candidates are required to attempt two questions each from Section A & B Section C will be compulsory.**

Section – A

Q.1 Find the Fourier series expansion of the following periodic function of period 4, where

$$f(x) = \begin{cases} 2 + x, & -2 \le x \le 0 \\ 2 - x, & 0 < x \le 2 \end{cases}.$$

Q.2 Find a series of sines and cosines of multiples of x which represents $x + x^2$ in $(-\pi, \pi)$.

Q.3 Express $\sin x$ as a cosine series in $[0, \pi]$.

Q.4 If f is bounded and integrable on $[-\pi, \pi]$ and a_n, b_n are its Fourier coefficients, then show that $\sum_{n=1}^{\infty}(a_n^2 + b_n^2) \le \frac{1}{\pi}\int_{-\pi}^{\pi} f^2\, dx$.

6x2 = 12

Section – B

Q.5 Find the Laplace transform of the following functions:

(i) $f(t) = [t],\ t \ge 0,$

(ii) $g(t) = |t-1| + |t-2| + |t-3|,\ t \ge 0.$

Q.6 State and prove Convolution theorem.

Q.7(a) Find $L(f(t))$ by using definition, where $f(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}.$

(b) Find $L^{-1}\left\{\frac{1}{(s-1)^2(s+5)}\right\}$.

Q.8 Apply Heaviside's expansion formula to find inverse Laplace transform of $\frac{1}{s(s^2+9)}$.

6x2 = 12

Section – C

Q.9(a) If $f(x)$ is an odd function, then find the value of a_n (Fourier coefficient) over the interval $[-l, l]$.

(b) Find the Fourier sine series of the function $f(x) = 1, 0 \le x \le 2$.

(c) Show that if *f(x)* is an even function then its Fourier expansion contains only cosine terms.

(d) Show that $\int_a^{a+2\pi} \sin mx \cos nx\, dx = 0$, if $m \ne n$.

(e) Evaluate $L(5e^{-2t} + 3\cos t - 5t^3)$.

(f) Evaluate $L^{-1}\left\{\frac{5s-8}{4s^2+36}\right\}$.

(g) State Heaviside's expansion formula.

(h) State and prove change of scale property of Laplace transforms.

8x2 = 16