ANALYSIS-I

Paper-II

(Semester-III)

B/2110 10663/NH

Time: Three Hours Maximum Marks: 40

Note: Attempt five questions in all. Select two questions each from Section A and B while Section C is compulsory.

SECTION-A

I. (a) Prove that every sequence contains a monotone subsequence. 4

(b) Prove that $\lim_{n\to\infty}\left[\left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}} = e.$ 2

II. (a) Show that the sequence $\{x_n\}$ defined by $x1=\sqrt{2}$, $x_n=\sqrt{2x_{n-1}}$ converges to 2. 4

(b) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is bounded. 2

III. (a) State and prove Gauss's Test. 4

(b) Discuss the convergence or divergence of the series

$\Sigma(\sqrt{n^2+1}-n)$. 2

IV. (a) Using Cauchy's Integral Test, discuss the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^p}, p>0$. 3

(b) Discuss the convergence of the series $\sum\frac{(n!)^2}{(2n)!}x^n, x>0$ 3

SECTION-B

V. (a) Prove that every continuous function is Riemann Integrable. 3

(b) If a function f is R-integrable on [a, b], then f2 is also R-integrable on [a, b]. 3

Contd

VI. (a) Let f(x)=3x+1 on [1,2]. Prove that f is R-integrable on [1,2] and $\int_1^2 (3x+1)dx = \frac{11}{2}$ 6

VII. If f is bounded function defined on [a, b], then for every ε >0,however small, there exists δ>0 such that $L(P,f) > \underline{\int_a^b} f - \varepsilon$ and $U(P,f) < \overline{\int_a^b} f + \varepsilon$ for all partitions with $||P|| < \delta$.

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VIII. (a) Let f be a function of bounded variation on [a, b]. If x ∈[a, b],let V(x)=V_f (a, x) and V(a)=0. Then every point of continuity of f is also a point of continuity of V. The converse is also true.

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SECTION-C

IX. (a) Show that $\lim_{n\to\infty} \frac{\log n}{n} = 0.$

(b) Define Bounded and Unbounded sequence.

(c) Is $\sum(-1)^{n-1}\frac{n+1}{n}$ convergent? justify your answer.

(d) Define Leibnitz's Test.

(e) Prove that if f(x) is an odd function then $\int_{-a}^{a} f(x)dx = 0.$

(f) Show that $\int_{-1}^{1}([x]-x)dx = -1,$ where [x] stands for integral part of x.

(g) Define Monotonic function.

(h) Define Rectifiable curve and its Arc Length.

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2 x 8 = 16