Roll No. .... **Total Pages : 3**

10398/NH

CS/2110

NUMBER THEORY-I

Paper-III

Option-(ii)

Semester-V

Syllabus-(Dec-19)

Time allowed : 3 Hours] [Maximum Marks : 40

Note: The candidates are required to attempt two questions each from Section A and B. Section C is compulsory.

SECTION-A

1. Given any integers a and b with $a > 0$, there exists unique integer q and r such that

   $b = aq + r, 0 \quad r < a$ 6

2. (i) State and prove Euler-Fermat theorem. 3

   (ii) Prove that for every integer n, n 2(mod 3) or $n^2 = n(\text{mod } 6)$. 3

3. State and prove Chinese remainder theorem and use it to find the least positive integer that give remainder 1, 2, 3 when divided by 3, 4, 5 respectively. 6

4. (i) Show that $n^{12} - a^{12}$ is divisible by 91 if n and a are prime to 91. 3

   (ii) State and prove Wilson's theorem. 3

SECTION-B

5. (i) State and prove Mobius Inversion Formula. 3

   (ii) Find all the solution of (x)=24 3

6. State and prove Quadratic Reciprocity Law and find the value of $\left(\frac{17}{19}\right)$ 6

7. Apply both Jacobi's and Legendre symbol to determine whether the following congruences have a solution

   (i) $x^2 \quad 135\,(\text{mod } 173)$ (ii) $x^2 = 21(\text{mod } 253)$ 6

8. State and prove Euler's Criterion. 6

SECTION-C

9. (i) State Fundamental theorem of Arithmetic.

(ii) Show that no integer in the sequence 11, 111, 1111, 11111, ... Is a perfect square.

(iii) Find the remainder when 2^{50} is divided by 7.

(iv) For m > 0 and a, b, c, d Z such that a b (mod m) and c d (mod m) then a + c b + d (mod m).

(v) ind ab ind a + ind b [mod (m)].

(vi) Find the values of d(180) and (180).

(vii) Prove that (n) (n+l) (n+2) (n+3) = 0 for any positive integer n.

(ix) Define Legendre's symbol 8×2=16